Lagrangian descriptors and regular motion

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Lagrangian descriptors introduced a decade ago have revealed as a powerful tool to unveil the intricacies of the phase space of dynamical systems in a very easy way [1]. They have been extensively used to study chaotic motion in a variety of different situations [2, 3, 7], but much less attention has been paid to applications to the regular regions of phase space. Recently J. Curbelo *et al.* have provided a way to locate the regular zones (invariant tori) [5, 6]. In this contribution, we show the potential of this recent mathematical tool, when suitably manipulated, to compute and fully characterize invariant tori of generic systems [8]. To illustrate the method, we present an application to the well known Hénon-Heiles Hamiltonian, a paradigmatic example in nonlinear science [4]. In particular, we demonstrate that the Lagrangian descriptors associated with regular orbits oscillate around an asymptotic value when divided over the integration time, which enables the computation of the frequencies of invariant tori.

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