A model for pattern formation in coral reefs

Miguel lvarez-Alegra, Manuel A. Matas and Dami Gomila
Instituto de Fsica Interdisciplinar y Sistemas Complejos, IFISC (CSIC-UIB), E-07122 Palma de Mallorca

Coral reefs are one of the most fascinating ecosystems in the world. The huge variety of living species they host together with the complex and colorful structures they form have made to gain for themselves the title of "rain forests of the sea". Even without considering interactions with the whole reef wildlife, corals biology is plenty of complex phenomena such as the symbiosis with microalgae named *Zooxanthellae* and emergent properties such as synchronized sexual events under very specific conditions or the self-organization of single polyps into well defined structures such as: massive, branching and table corals. Although there are some "lonely" species, coral polyps typically group into colonies of the same species forming the previously mentioned structures. Colonies living in the same region constitute a reef. Various species of these animals (corals) synthesize aragonite (a phase of calcium carbonate) during their clonal growth to build themselves a hard exoskeleton. When an entire colony dies, another new polyp can settle over the hard structure left behind by the dead colony and start a new colony on top of these remains. This process is repeated over centuries leading to the formation of large aragonite structures which display some recognizable patterns that can be spotted around the world. These include closed atolls and parallel stripes, together with large groups of closed atolls and little halos inside these big atolls (an example of self-similarity in this system).

In this work, we address the study of pattern formation in coral reefs by proposing a set of differential equations that govern the spatiotemporal evolution of some variables describing the system. Previous attempts on this direction include the work of Mistr and Bercovici [1], in which they successfully describe the formation of parallel stripes, and some models including reaction - fractional diffusion schemes [2], based on the previous work of Mistr and Bercovici. Some other authors have been proposed a model based on the chemistry of the aragonite synthesis combined with diffusion [3]. However, although these works reproduce some of the observed patterns in nature, they do not provide a whole description of the system and they do not include they key role of hydrodynamics in reef shaping.

We propose a new model to describe coral reefs by considering four fields in two spatial dimensions corresponding to: alive coral tissue P, dissolved nutrients in ocean water N, amount of accumulated aragonite A and the saturation state of aragonite in ocean water Ω. The amount of accumulated aragonite A is easily converted into the height the reef has reached at time t and we consider that at every moment live corals are settled over this hard rock structure or over the sea floor if there is no hard rock. These four fields interact following a reaction-diffusion model (plus an advective term for water flux) that includes terms representing birth, death, clonal growth and nutrients consumption for the polyps, together with some effects due to the height the reef has reached. Reef erosion and aragonite synthesis with its consequent alteration in the saturation state of aragonite in ocean water have also been included. After calibrating the parameters of the model based on real data and some reasonable assumptions, we have started to study the model's behavior in one dimension, obtaining some preliminary results that resemble the shape of closed atolls (Fig. 1).

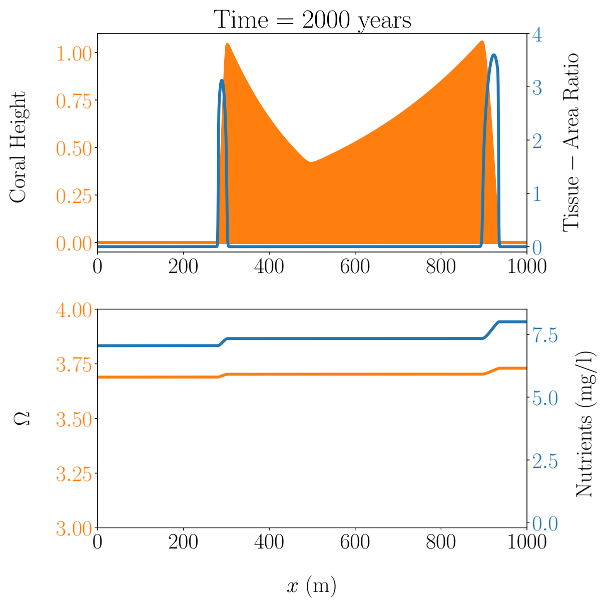

Fig. 1. Initial gaussian distribution of polyps and coral height placed at $x = 500$ m after 2000 years of evolution. Water flux coming from the right ($x = 1000$ m) flowing to the left. Blue curve at the panel above represents the ratio between alive coral tissue area respect to the sea floor area at each point. Orange shaded curve represents the height that the solid structure of aragonite rock has reached, normalized to the height of water column measured from the sea floor to the sea level. Blue and orange curves in the panel below represent nutrients concentration and aragonite saturation state, respectively.

[1] Mistr, S. and Bercovici, D. *A Theoretical Model of Pattern Formation in Coral Reefs*, Ecosystems **6** 0061-0074 (2003).

[2] Somathilake L. W. and Burrage, K. *A Space-Fractional-Reaction-Diffusion Model for Pattern Formation in Coral Reefs*, Cogent Mathematics & Statistics **5** (2018).

[3] Nakamura, T. and Nakamori, T. *A geochemical model for coral reef formation*, Coral Reefs **26**, 741755 (2007).