On the mean square displacement of intruders in freely cooling granular gases

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We compute the mean square displacement (MSD) of intruders immersed in a freely cooling granular gas composed of inelastic smooth hard spheres ("grains") [1]. In general, intruders and grains are assumed to have different mechanical properties, implying that non-equipartition of energy must be accounted for in the computation of the diffusion coefficient D. In the hydrodynamic regime, the time decay of the granular temperature T of the cooling gas is known to be dictated by Haff's law [2]; the concomitant decay of the intruder's collision frequency entails a time decrease of D. Explicit knowledge of this time dependence allows us to determine the MSD by integrating the corresponding diffusion equation.

Previous studies have found a logarithmic time dependence of the MSD in the limit cases when intruders and grains are mechanically equivalent [selfdiffusion] (see e.g. [3, 4]) or when the intruder's mass is much larger than that of the grains [Brownian limit] (see [5]). We find that the logarithmic time dependence of the MSD extends beyond the two aforementioned cases, and holds in all spatial dimensions for arbitrary values of the mechanical system parameters (Euclidean spatial dimension d, intruder-grain mass ratio m_0/m , intruder-grain diameter ratio σ_0/σ, and the respective coefficients α_0 and α of normal restitution for intruder-grain collisions and grain-grain collisions). The ultraslow type of diffusion observed here (slower than any power-law in time) is due to the energy loss in every collision, as opposed to other systems where it arises from the spatial disorder of the medium.

Beyond the self-diffusion and Brownian cases, we carry out a comprehensive study of the MSD dependence on the mechanical system parameters. For a proper choice of the latter, interesting features emerge from our analysis, such as a non-monotonic dependence of the MSD on the restitution coefficients and on the intruder-hard sphere mass ratio (see Fig. 1). To understand the observed behaviour, we analyze in detail the properties of the intruder's ballistic displacements inside the granular gas.

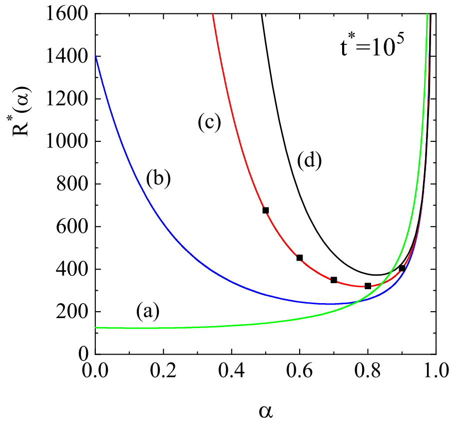

Fig. 1. Plot of the MSD scaled with the squared mean free path ℓ^2 of the three-dimensional granular gas $R^* \equiv \langle|\Delta\mathbf{r}|^2\rangle/\ell^2$ as a function of the coefficient of normal restitution $\alpha = \alpha_0$ for $\sigma_0/\sigma = 2$ and four different values of the mass ratio: $m_0/m = 1/2$ (a), $m_0/m = 5$ (b), $m_0/m = 8$ (c), and $m_0/m = 10$ (d). The symbols refer to the results obtained from Monte Carlo simulations for $m_0/m = 8$. All the curves refer to the same value $t^* = 10^5$ of the scaled time $t^* \equiv \nu(0)^{-1}t$, where $\nu(0)$ is the initial value of the average grain-grain collision frequency and t denotes physical time.

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