Dynamics of hard particles confined by an isotropic harmonic potential

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The dynamics of a system composed of elastic hard particles confined by an isotropic harmonic potential is studied. In the low-density limit, the dynamics is described by the Boltzmann equation and the system does not reach equilibrium except for some particular class of initial conditions. On the contrary, the system reaches a periodic in time state in which the velocity distribution function is Gaussian, but with the hydrodynamic fields oscillating in time with some specific profiles. It is shown that this so-called *breather* state is completely specified by the constants of the motion, the mean square displacement, $\langle r^2 \rangle$, at the initial time and its derivative with respect to time also at the initial time. This is due to the fact that, at this level of description, $\langle r^2 \rangle$ verifies a closed second order differential equation. For low but finite densities, the dynamics of the system is analyzed by taking into account the finite size of the particles. Under well-controlled approximations, a closed evolution equation for $\langle r^2 \rangle$ is derived, obtaining that it decays to its equilibrium value, oscillating with a frequency slightly modified with respect to the Boltzmann values. The time average of the oscillations is also renormalized. An excellent agreement is found between Molecular Dynamics simulation results and the theoretical predictions for the frequency and the time average of the oscillations. For the relaxation time, the agreement is not as good as for the two previous quantities and the origin of the discrepancies is discussed.