Random geodesics and the boundary of KPZ

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Metric geometry has a large number of applications in physics. Specifically, random metric spaces present useful methods to obtain effective descriptions of a large number of phenomena showing fluctuations with a geometric origin. For example, thermal fluctuations of important biophysical objects such as fluid membranes. In the context of random metric spaces, a model of interest is given by first-passage percolation (FPP), which was originally introduced as a model of fluid through a random medium.

The FPP model is defined as follows. Given an undirected discrete lattice, we place a non-negative random variable τ_e on each nearest-neighbor edge, which is interpreted as a passage time. The collection $\{\tau_e\}$ is assumed to be independent and identically distributed, with common distribution F. The main objects of interest are the shortest-time paths between any pair of lattice nodes, also called geodesics, and the balls $B(T)$, i.e. the set of nodes which can be reached in a time less than the passage time T. This ball has been studied on the context of random metrics showing, under some circumstances, the exponents corresponding to the Kardar-Parisi-Zhang universality class (KPZ) [1].

These geometric properties were studied by our group in the context of a discrete space using the FPP model [2]. Two types of planar lattices have been considered: regular lattices and disordered lattices. For the regular case we considered square lattices and we studied the fluctuations of the times of arrival to points along the axis and the diagonal directions. For the axis direction, we can observe a pre-asymptotic regime while in the diagonal direction this pre-asymptotic regimen is not present. In order to understand this difference, the concept of geodesic degeneracy is introduced, i.e. the number of geodesics joining the given points in absence of noise. Afterwards, an analogous analysis was done on *Delaunay lattices* as a relevant example of disordered lattices. In both types of lattices, a complete characterization of the scaling behavior has been performed with the study of the lateral deviation of the geodesics, defined as the Euclidean distance from its middle point to the straight line joining the end points. This morphological property is related to the correlation length to the underlying surface. Finally, a complete characterization of the fluctuations of the arrival times and the geometry of the geodesics was performed, allowing us to conjecture that the FPP model belongs to the KPZ universality class under some mild conditions, as supported by our numerical results.

Subsequently, the relevance of the lateral deviation on the emergence of the KPZ universality class has been studied. In order to do that, we have set up an artificial constraint for the lateral desviation of the ensemble of geodesics, by imposing a curved boundary on the underlying manifold, with intention of proving the stability of the KPZ universality class (Fig.1).

This study has been done for the diagonal direction, showing the relevance of the geodesic degeneracy in the pre-asymptotic regime which we can find in the axis. Moreover, our numerical experiments suggest a continuous deformation of the scaling exponents and the arrival time distribution as our boundary constraints more and more the space allowed for the geodesics, thus proving that KPZ scaling is lost when the geodesics are not given enough lateral space.

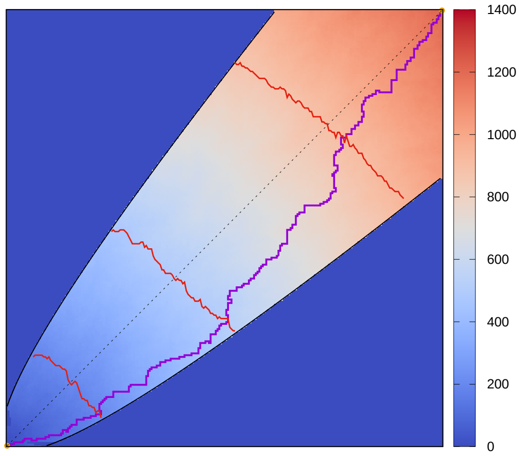

Fig. 1. A geodesic along the diagonal direction of a square lattice under a boundary constraint of the form $f(x) \propto r^{\alpha}$. Differents colors represent different arrival times, and some isochrones can be observed.

[1] Silvia N Santalla et al 2015 *New J. Phys.* **17** 033018

[2] Pedro Córdoba-Torres et al *J. Stat. Mech.* (2018) 063212