Non-Markovian random walks characterize network robustness to non-local cascades

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Understanding the interplay between structure and dynamics is still one of the major challenges in network science. A central question concerns the robustness of a system against perturbations, since it can advance the development of powerful analytical techniques to explain and unravel rich phenomenology, as well as it can provide a solid ground for informed interventions.

A main assumption behind the analysis of robustness is that for a system to be functional, it needs to be connected. Hence, concepts and techniques from percolation theory become useful and are frequently employed. This is a completely static approach, where a fraction of nodes (or links), either selected uniformly at random or based on topological or non-topological descriptors, is removed from the network. From a dynamical point of view, small failures placed in the network may evolve —according to some rules that depend on the phenomenon one is trying to model— causing system-wide catastrophic cascades. For the sake of mathematical tractability, cascades are assumed to spread via direct contacts. However, be it because the physical mechanisms behind the failure propagation permit far-off malfunctions, be it because the knowledge on the observed network topology is incomplete and the failure propagates through hidden or unobserved edges, real-world cascades display non-local features. From a modeling standpoint, some mechanisms like flow redistribution can lead to non-local spreading of failures but the mathematical treatment has been hitherto under-researched due to its sophistication and there is no direct way to control the underlying properties of the non-local events, seriously undermining our understanding of the phenomenon.

To better reconcile theory and observations, we propose a dynamical model of non-local failure spreading that combines local and non-local effects. We assume that the cascade unfolds in a timescale much faster than the recovery of nodes, and that a disrupted unit cannot be visited more than once by the failure. This fact causes the failure to be no longer Markovian and, for modeling purposes, a natural choice is to consider a Self-Avoiding Random Walk-like (SARW) dynamics on the network. To cope with the non-locality, we introduce a teleporting probability: at each step t the failure proceeds as in a SARW —uniformly choosing an operational neighbor and transitioning there— with probability $1-\alpha \in [0,1]$, otherwise with probability α it teleports to any operative node according to a teleporting rule $T_t(k)$, time- and degree-dependent. We name this the self-avoiding teleporting random walk (SATRW), which interpolates between percolation (purely non-local process, $\alpha = 1$) and the growing SARW (purely local process, $\alpha = 0$).

We have characterized the rich critical behavior of our model by providing analytical expressions for several quantities employed to assess the systems robustness, such as the time-dependent degree distribution $p_t(k)$, the size of the giant component in the residual network as the process evolves s_t, the cascade first-stop time distribution, and the mean value of $S^{(\text{STOP})}$, the giant component at the cascade stop. These robustness descriptors display an excellent agreement with simulations in synthetic systems characterized by different types of complexity in terms of the heterogeneity of their structural connectivity. We find remarkable differences between homogeneous and heterogeneous systems, e.g., their dependence, or lack thereof, on the particular network parameters. However, we also report some hidden similarities between them, such as a dynamical version of the popular *robust-yet-fragile* feature to static attacks. It is worth noticing that, despite our framework is expected to work for locally tree-like networks lacking topological correlations, such as degree-degree ones, it still works in empirical settings as we have shown for the case of a biomolecular system, namely the interactome of the nematode *C. elegans*, and an infrastructural system, namely a national air traffic network, shown in Fig. 1.

Our findings provide a solid ground for the analytical study of network robustness, in particular, and for non-local non-Markovian processes, in general. The article is currently under review.

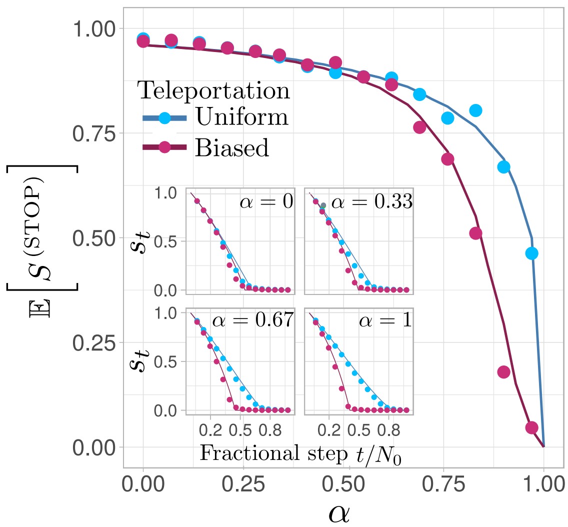

Fig. 1. Expected value of size of the giant component at the cascade stop as a function of the teleportation parameter α for the air traffic network. In the insets, evolution of s_t as a function of the fractional time t/N_0, for different values of α. Solid lines come from theory, markers from simulations. Two type of teleporting rules are shown.