Casimir forces on curved backgrounds

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The quantum vacuum on a static space-time is nothing but the ground state of a certain Hamiltonian. Therefore, it is subject to quantum fluctuations which help minimize the energy. Yet, these fluctuations are clamped near the boundaries, giving rise to the celebrated *Casimir effect* [1, 2].

For fields subject to conformal invariance, the Casimir force is associated to the conformal anomaly. Using open boundary conditions on a $(1+1)D$ system with size N, the energy of the ground state can be proved to be [3]

$$E(N) = \epsilon_1 N + \epsilon_0 + \frac{cv}{6N} + O(N^{-2}), \tag{1}$$

with ϵ_0 and ϵ_1 constants, v standing for the Fermi velocity, and c is the central charge. Let us also remark the fact that the sign of the force can be changed from attractive to repulsive by suitable choice of the boundary conditions.

We characterize Casimir forces for the Dirac vacuum on free-fermionic chains with smoothly varying hopping amplitudes, which can be seen to correspond to (1+1)D curved space-times with a static metric of optical type in the thermodinamic limit. The metrics considered are anti-de Sitter (Rainbow system), Rindler, oscillatory and random [4]. Thus, our main objective is to characterize how Eq. (1) changes in presence of a static graviational field for fermionic (1+1)D systems.

Let us consider a fermionic chain of N sites, whose dynamics is described by the Hamiltonian

$$H_N(\mathbf{J}) = -\sum_{m=1}^{N-1} J_m c_m^\dagger c_{m+1} + \text{h.c.}, \tag{2}$$

where $\mathbf{J} = \{J_m\}_{m=1}^{N-1}$ are the *hopping amplitudes*, $J_m \in R^+$ referring to the link between sites m and $m+1$, and $c_m^\dagger$, c_{m+1} are the fermionic creation and annihilation operators on sites m and $m+1$, respectively. Hamiltonian (2), which is quadratic in the fermionic operators, is also called free fermion Hamiltonian and is solvable in terms of single-body sates. In the thermodynamic limit, if the $\mathbf{J}$ are smooth, Eq. (2) corresponds to the Dirac field on a metric of the form [5]

$$ds^2 = -J^2(x)dt^2 + dx^2. \tag{3}$$

Moreover, when we move away from half-filling a new phenomena is observed. A depletion region appears to arise in the fermionic density and in next-neighbours correlators as well.

We have considered the continuum limit of Hamiltonian (2) with

$$\Psi = \Psi_L + \Psi_R, \tag{4}$$

$$c_m = \sqrt{a}\left(e^{ik_F x}\Psi_L(x) + e^{-ik_F x}\Psi_R(x)\right), \tag{5}$$

where k_F is the Fermi momenta.

Expanding the fields, $\Psi_L(x)$ and $\Psi_R(x)$, to second order we end up with a new Hamiltonian which equations of motion give rise to a Schrödinger-like expression where an effective potential could explain the depletion phenomena. It is important to say that away from half filling the theory is not conformal.

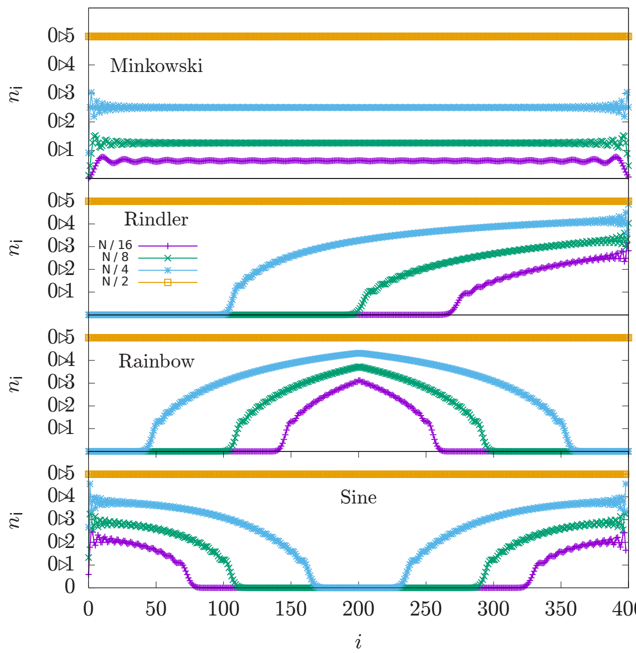

Fig. 1. Fermionic density for different filling-factors with $N = 400$ sites. Four metrics have been considered: Minkowski, Rindler, Rainbow and Sine.

[1] H.B.G. Casimir, D. Polder, Phys. Rev. **73**, 360 (1948).

[2] G.L. Klimchitskaya, V.M. Mostepanenko, Contemp. Phys. **47**, 131 (2006).

[3] P. Di Francesco, P. Matthieu, D. Sénéchal, *Conformal Field Theory*, Springer (1997).

[4] B. Mula, S. N. Santalla and J. Rodríguez-Laguna, *Casimir forces on deformed fermionic chains*, Phys. Rev. Res. **3** (2021) 013062(9).

[5] J. Rodríguez-Laguna, L. Tarruell, M. Lewenstein, A. Celi, Phys. Rev. A **95**, 013627 (2017).