# Finite-time scaling for epidemic processes with power-law superspreading events

Carles Falcó[1], Álvaro Corral[2,3,4],
[1]Mathematical Institute, University of Oxford, OX2 6GG Oxford, United Kingdom
[2] Centre de Recerca Matemàtica, Edifici C, Campus Bellaterra, E-08193 Barcelona, Spain
[3] Departament de Matemàtiques, Facultat de Ciències, Universitat Autònoma de Barcelona, E-08193 Barcelona, Spain
[4] Complexity Science Hub Vienna, Josefstädter Stra$\beta$e 39, 1080 Vienna, Austria

Epidemics unfold by means of a spreading process from each infected individual to a random number of secondary cases. It has been claimed that the so-called superspreading events in COVID- 19 are governed by a power-law tailed distribution of secondary cases, with no finite variance. Using a continuous-time branching process, we show that for such power-law superspreading the survival probability of an outbreak as a function of time and the basic reproductive number fulfills a finite- time scaling law (analogous to finite-size scaling) with universal-like characteristics only dependent on the power-law exponent. This clearly shows how the phase transition separating a subcritical and a supercritical phase emerges in the infinite-time limit (analogous to the thermodynamic limit). We quantify the counterintuitive hazards infinite-variance superspreading poses and conclude that superspreading only leads to new phenomenology in the infinite-variance case.

This work builds on existing literature studying branching process models and their universality [1, 2, 3]. We focus on offspring distributions with power-law decay with an exponent $\gamma$ between 2 and 3, which makes the variance of the distribution divergent. In this context, the mean of the offspring distribution $R_0$ becomes of limited utility as we cannot define an associated standard error to it. While the average dynamics of the branching process are independent on higher moments of the offspring distribution, its variance does play an important role in the dynamics of the process. More, and in contrast with the finite variance case studied in previous works, a new universality class appears for every value of the exponent of the power-law. We focus on the study of the survival probability of the process $q(t)$, which gives direct insights in quantities such as the expected duration of an outbreak, and the total number of contagions. In particular, and as mentioned above, we find that near criticality ($R_0$ close to 1) a finite-time scaling law (analogously to finite-size scaling in the theory of phase transitions) emerges making explicit the dependency on time of the process. Such scaling law reads

$$q(t) \propto \frac{G_\gamma(z)}{t^{1/(\gamma-2)}} \tag{1}$$

with the $\gamma-$dependent scaling function

$$G_\gamma(z) = \left(\frac{ze^z}{e^z-1}\right)^{\frac{1}{\gamma-2}}, \tag{2}$$

and $z \propto (R_0-1)t$, a rescaled distance to the critical point.

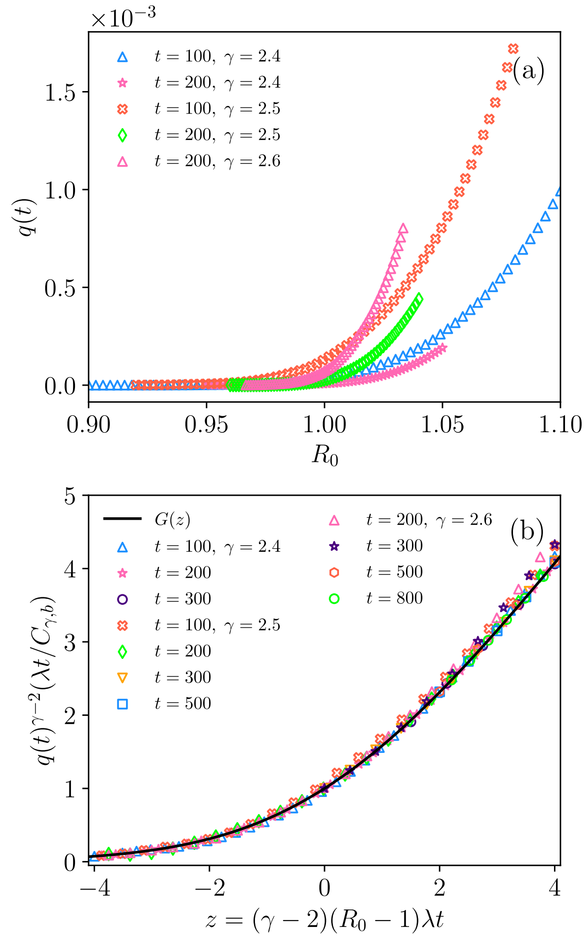


Fig. 1. (a) Survival probabilities $q(t)$ versus $R_0$. (b) General rescaling of $q(t)^{\gamma-2}$ given by Eq. (1).

[1] R. Garcia-Millan, F. Font-Clos, and A. Corral. *Finite-size scaling of survival probability in branching processes*, Phys. Rev. E, 91:042122, 2015.

[2] N. Wei and G. Pruessner. *Comment on "Finite-size scaling of survival probability in branching processes"*, Phys. Rev. E, 94:066101, 2016.

[3] R. Garcia-Millan, J. Pausch, B. Walter, and G. Pruessner. *Field-theoretic approach to the universality of branching processes*, Phys. Rev. E, 98:062107, 2018.