Correlation lag times provide a reliable early-warning indication of approaching bifurcations in spatially extended dynamical systems

Giulio Tirabassi and Cristina Masoller
Departament de Física, Universitat Politècnica de Catalunya, Rambla Sant Nebridi 22, 08222 Terrassa, Spain

Identifying upcoming bifurcations and regime transitions from observations is a significant challenge in time series analysis. Well-known early warning indicators (EWIs) are the increase in spatial and temporal correlation. Here we propose a new indicator that takes into account both. By inspecting the distribution of lag times that maximize the cross-correlation between pairs of adjacent points, we find that the variance of the distribution consistently decreases as the bifurcation approaches. We demonstrate the reliability of this indicator using different models that exhibit different types of bifurcations [1].

Formally, we define the lag, τ_{ij}, that maximizes the absolute cross-correlation between two time series u_i and u_j, as

$$\tau_{ij} = \operatorname{argmax}_{\substack{\tau \\ 1\leq\tau\leq\tau_{\max}}} \left(\left| \sum_t u_i(t) u_j(t+\tau) \right| \right), \qquad (1)$$

where u_i and u_j are normalized to have zero mean and unitary variance. From an operational standpoint, we search for the maximum in a range of τ values from 1 up to a maximum value, $\tau_{\max}$, that is 4% of the length of the time series, and computed the cross-correlation using 96% of the time series length. Moreover, we limit the calculation only to ij nodes that are first neighbors. To test the variance of τ_{ij}, σ_τ, as a possible EWI, we simulated three different models where different types of bifurcations occur. By studying how σ_τ varies when the parameter approaches the bifurcation, we obtain integrated spatio-temporal information, and we show that it is more informative than the autocorrelation or spatial correlation.

We report in Fig. 1 an example for one of the models we studied. We consider a model that describes the evolution of a scalar field, u, in a 1D space, with diffusion, noise, and a bistable potential whose asymmetry is governed by the bifurcation parameter, α.

$$\frac{\partial u}{\partial t} = -u^3 - \alpha u^2 + u(1+\alpha) + D\frac{\partial^2 u}{\partial x^2} + \xi \qquad (2)$$

where D modulates the diffusion strength, and ξ is a gaussian noise term. Depending on α, the system has one or three fixed points. When α approaches a critical value, two fixed points collide giving rise to a transcritical bifurcation. Approaching the bifurcation point, the time series variance increases and CSD is detected through lag-1 correlation and spatial correlation (Fig. 1a). However, although both these quantities increase, their behavior does not reveal how close the system is to the bifurcation, since they grow monotonically and the bifurcation occurs way before they reach their maximum possible value. In contrast, σ_τ displays a non-monotonic behavior with respect to the bifurcation parameter (Fig. 1b). In particular, approaching the bifurcation, σ_τ slightly increases collapsing right before the transition. The variation of σ_τ is more informative than that of the mean value of τ_{ij} distribution (dashed line), as the maximum provides a condition to be fulfilled for the transition to happen. The inflection point can be explained making the hypothesis that τ_{ij} distribution is formed by two components: one localized at low τ_{ij} that dominates when close to the bifurcation, and a uniform distribution that dominates when the system is far from it. The bifurcation parameter moves the distribution towards these two extremes. The maximum represents the moment in which the localized distribution overtakes the random one.

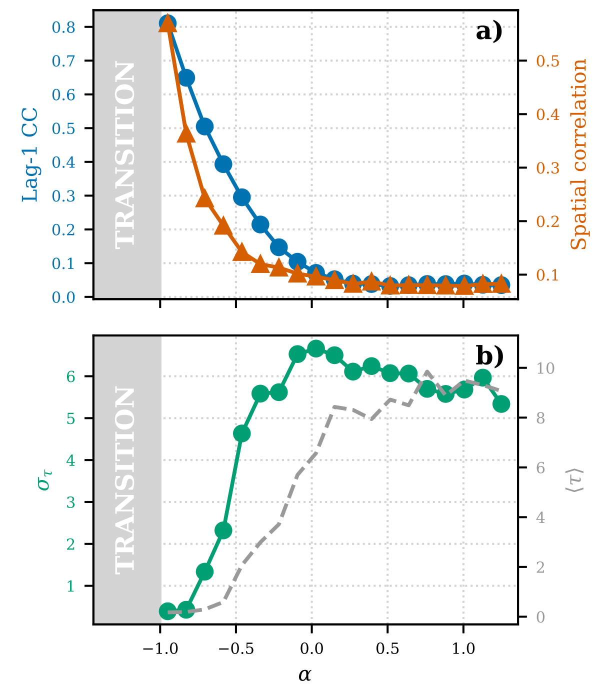

Fig. 1. **(a)** Conventional EWIs, lag-1 temporal correlation and spatial cross-correlation, as a function of the bifurcation parameter, α, for the scalar bistable 1D model. **(b)** Mean, $\langle\tau\rangle$, and standard deviation, σ_τ, of the distribution of lags that maximize the cross-correlation. One single realization of the dynamics has been used.

[1] G. Tirabassi and C. Masoller, "Correlation lags give early warning signals of approaching bifurcations", Chaos, Solitons & Fractals 155, 111720 (2022).