Analyticity constraints bound the decay of the spectral form factor

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Quantum mechanics and non-linear dynamics are two of the most prominent scientific triumphs of the 20th century. The connection between them, i.e. the study of chaotic behaviour of quantum systems, also known as *quantum chaos*, is an exciting field of study which gathers the high-energy, quantum-information and statistical mechanics communities.

The modern study of quantum chaos relies on dynamical quantities like the Out-of-Time-Order Correlator (OTOC) and the Spectral Form Factor. The former measures how much two operators fail to commute at different times and it allows for a definition of a *quantum lyapunov exponent*, while the latter is the Fourier transform of the two-level correlation function and characterises chaos in the *correlation hole* which serves as a signature of quantum chaotic dynamics. Recently a universal bound on the quantum Lyapunov exponent was conjectured $\lambda \leq 2\pi/(\beta\hbar)$ [1, 2, 3] which essentially states that the maximum rate at which quantum chaos can develop is proportional to the temperature of a system at equilibrium. This 'MSS-bound' on the Lyapunov exponent is inspired by black hole physics and AdS/CFT correspondence but its proof relies on a mathematical property of the OTOC, namely that it is analytic in some half-stripe of the complex plane when we consider complex times $t \to t + i\tau$.

Our main observation is that the mathematical machinery developed for proving the MSS bound is not restricted to the OTOC or even to chaos itself. Therefore it also yields universal bounds on different dynamical quantities. Particularly, we find that a dynamical quantity f_t that is (i) analytic on a half-stripe of the complex plane when $t \to t + i\tau$, (ii) normalized such that $|f_{t+i\tau}| \leq 1$ and (iii) exponentially decaying or growing, can be bounded by analyticity.

Specifically, we find that the Spectral Form Factor obeys all of these properties in its intermediate-time exponential decay $S(\beta, t) \sim e^{-\eta t}$, before the chaotic features show up in the correlation hole, therefore yielding a universal bound which does not depend on chaos. The bound we find on the *inflection exponent* η is given by

$$\eta \leq \frac{\pi}{2\beta\hbar}. \tag{1}$$

We illustrate the derived bound in systems with non-chaotic and chaotic dynamics. The former case is exemplified with the quantum harmonic oscillator while for the latter we use an ensemble from Random Matrix Theory. To bridge between the two cases we use the *quantum kicked top*, a model proposed in the original studies of quantum chaos which describes an angular momentum subject to a free precession and some periodic non-linear kicks [4]. The inflection exponent for this system shows a similar qualitative behaviour in both dynamical regimes (see Fig. 1). Furthermore, the exponent gets very close to the bound in both regimes, while the MSS bound is saturated only for *maximally chaotic* systems. This supports the idea that the derived bound is not sensitive to chaos and therefore bounds based on analyticity can apply to a wider range of quantities, not necessarily within the context of quantum chaos. Furthermore we also compare our derived universal bound with other physical bounds, like Quantum Speed Limits, and find that the derived universal bound can be tighter in some regimes than Quantum Speed Limits.

We believe that conditions (i)-(iii) are not so restrictive and could yield universal bounds in dynamical quantities throughout Quantum and Statistical Mechanics.

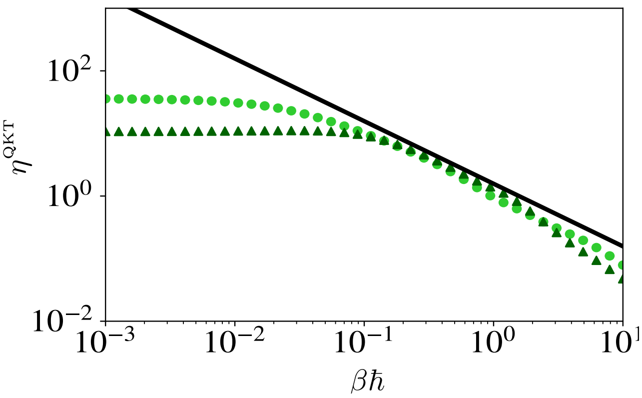

Fig. 1. **Inflection exponent for the Quantum Kicked Top** $\eta^{\rm QKT}$ for chaotic (light-green circles) and non-chaotic (dark-green triangles) dynamics along with the derived bound (1) (black solid line).

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