Transport coefficients of a granular gas immersed in a gas of elastic hard spheres

Rubén Gómez González[1] and Vicente Garzó[2]
[1]Departamento de Física, Universidad de Extremadura, Avda. de Elvas s/n, 06006 Badajoz (Spain)
[2] Departamento de Física, Instituto Universitario de Computación Científica Avanzada (ICCAEx), Universidad de Extremadura,, Avda. de Elvas s/n, 06006 Badajoz (Spain).

Let us consider an ensemble of inelastic hard spheres (granular gas) of mass m immersed in a gas of elastic hard spheres (molecular gas) of mass m_g. We assume that the granular gas is sufficiently rarefied and therefore the presence of the grains does not perturb the state of the surrounding molecular gas. In this situation, the molecular gas may be treated as a thermostat at a fixed temperature T_g. Nonetheless, although the number density of the grains is much smaller than that of the molecular gas, we account for the grain-grain collisions in its kinetic equation. Under these conditions, the extension of the classical kinetic theory of gases is considered as an suitable tool to model these multiphase systems.

In the low-density regime, the one-body distribution function $f(\mathbf{r},\mathbf{v},t)$ obeys the Boltzmann equation:

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla f=J[\mathbf{v}|f,f]+J_g[\mathbf{v}|f,f_g]. \qquad (1)$$

Here, the Boltzmann collision operator $J[\mathbf{v}|f,f]$ gives the rate of change of the distribution $f(\mathbf{r},\mathbf{v},t)$ due to binary *inelastic* collisions between granular particles characterized by a constant coefficient of normal restitution α. On the other hand, the Boltzmann–Lorentz operator $J_g[\mathbf{v}|f,f_g]$ accounts for the rate of change of the distribution $f(\mathbf{r},\mathbf{v},t)$ due to *elastic* collisions between granular and molecular gas particles. Since the molecular gas is assumed to be at equilibrium, its distribution function f_g is a Maxwellian distribution.

The Boltzmann–Lorentz equation Eq. (1) applies in principle for arbitrary values of the mass ratio m/m_g. However, one of the main aims of the present work is to analyze the conditions under which the transport properties derived from Eq. (1) reduce to that obtained using a Langevin-like model [1]. A careful analysis shows that both models overlap when the granular particles are much heavier than that of the interstitial gas (Brownian limit, $m/m_g\to\infty$). In this limit case, the operator $J_g[\mathbf{v}|f,f_g]$ can be approximated by the Fokker–Planck operator [2]:

$$J_g[\mathbf{v}|f,f_g]\to\gamma\frac{\partial}{\partial\mathbf{v}}\cdot\left(\mathbf{v}+\frac{T_g}{m}\frac{\partial}{\partial\mathbf{v}}\right)f(\mathbf{v}), \qquad (2)$$

where the drift or friction coefficient $\gamma\propto\sqrt{m_g/m}$ is a function of the parameters of the molecular gas [3].

On the other hand, we want to extend the results for the transport coefficients obtained in [1] to arbitrary values of the mass ratio m/m_g. To do so, the Chapman–Enskog-like expansion is applied to the Boltzmann kinetic equation. First step is to characterize the reference state in the perturbation scheme, namely the homogeneous steady state (HSS). In the HSS, the energy lost due to inelastic collisions is exactly compensated for by the energy injected by the collisions with the more rapid particles of the molecular gas. This simple situation allows us to compute the steady granular temperature T (see Fig. 1 for an example with a given value of the reduced bath temperature T_g^*) [4].

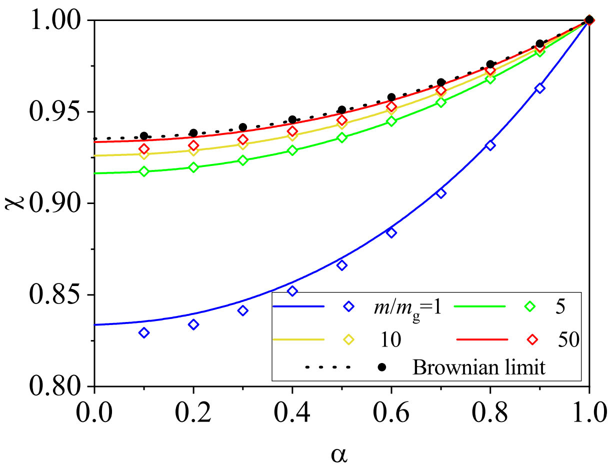

Fig. 1. Temperature ratio $\chi\equiv T/T_g$ versus the coefficient of normal restitution α for $T_g^*=1000$, and four different values of the mass ratio m/m_g. The solid lines are the theoretical results obtained by numerically solving Eq. (1) and the symbols are the Monte Carlo simulation results. The dotted line is the result obtained in [1] by using the Langevin-like suspension model (2).

Once the homogeneous state is characterized, the next step is to solve the Boltzmann equation by means of the Chapman–Enskog-like expansion to first order in spatial gradients. The Navier–Stokes–Fourier transport coefficients are explicitly determined by considering the leading terms in a Sonine polynomial expansion when steady-state conditions apply. In general, the transport coefficients show a complex dependence on the mass ratio m/m_g, the (reduced) bath temperature T_g^*, and the coefficient of restitution α. Moreover, in the Brownian limit ($m/m_g\to\infty$) we recover the results previously derived by using the Langevin-like model [1].

As an application, a linear stability analysis is performed showing that the HSS is always linearly stable regardless of the mass ratio considered.

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