Splitting-thermostat-engineered protocol for Mpemba effect in granular gases of inelastic and rough hard disks

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In the last few years, Mpemba effect (an initially further from equilibrium system may relax faster than that initially closer) has attracted much attention in statistical physics and complex systems, specifically in granular gas dynamics phenomenology [1].

In this work, we consider a monodisperse dilute granular gas of inelastic and rough hard disks of mass m, diameter σ, and moment of inertia $\frac{m\sigma^2}{4}\kappa$. We adopt the simplest two-parameter model with a constant coefficient of normal restitution ($0 < \alpha < 1$), parameterizing the inelasticity, and a constant coefficient of tangential restitution ($-1 < \beta < 1$), accounting for the roughness of the particles.

To compensate for the dissipation of energy (translational and rotational contributions), we assume thermalization through heating from a stochastic thermostat that could act directly on both, rotational and translational, classes of degrees of freedom. That is, the system relaxes by means of the action of a homogeneous stochastic volume force ($\mathbf{F}^{\mathrm{n}}$) and torque (τ^{n}) with the properties of a Gaussian noise,

$$\langle \mathbf{F}_i^{\mathrm{n}}(t)\rangle = \mathbf{0},\ \langle \mathbf{F}_i^{\mathrm{n}}(t)\mathbf{F}_j^{\mathrm{n}}(t')\rangle = \mathsf{I} m^2(1-\varepsilon)\chi_0^2\delta_{ij}\delta(t-t'), \tag{1}$$

$$\langle \tau_i^{\mathrm{n}}(t)\rangle = 0,\ \langle \tau_i^{\mathrm{n}}(t)\tau_j^{\mathrm{n}}(t')\rangle = \frac{m^2\sigma^2}{2}\kappa\varepsilon\chi_0^2\delta_{ij}\delta(t-t'), \tag{2}$$

where i, j are particle indices, I is the 2×2 unit matrix, χ_0^2 refers to the total intensity of the thermostat, and ε is the *fraction* split up from the total intensity acting on the particles spinning. This class of thermostats allows us to design a controlled protocol for observing Mpemba effect in a kinetic-theoretical context. Dimensional analysis suggests to define the reference temperature $T^{\mathrm{n}} \equiv m(\chi_0^2/n\sigma)^{2/3}$, where n is the number density, as a measure of the total intensity of the thermostat

In a recent work [2], the Mpemba effect was described for hard spheres (i.e., not disks) and $\varepsilon = 0$, concluding that an initially hotter system A would need initially a smaller rotational-to-translational temperatures ratio, θ, with respect to an initially colder system B to ensure a crossover between the granular temperature curves, $T(t)$. These initial conditions are here extrapolated to any splitting $0 \le \varepsilon \le 1$ and to a two-dimensional granular gas.

Assuming that the nonequilibrium velocity distribution function of the gas can be approximated by a time-dependent, two-temperature Maxwellian distribution, the dynamics of the system (at given T^{n} and ε) is described by a coupled set of equations for the quantities θ and T. As physically expected, the rotational-to-translational temperature ratio in the steady state (θ^{st}) increases monotonically as the fraction of the total intensity of the thermostat applied to the rotational degrees of freedom (ε) increases. Moreover, the steady-state granular temperature (T^{st}) is proportional to the reference temperature (T^{n}) associated with the total intensity of the noise.

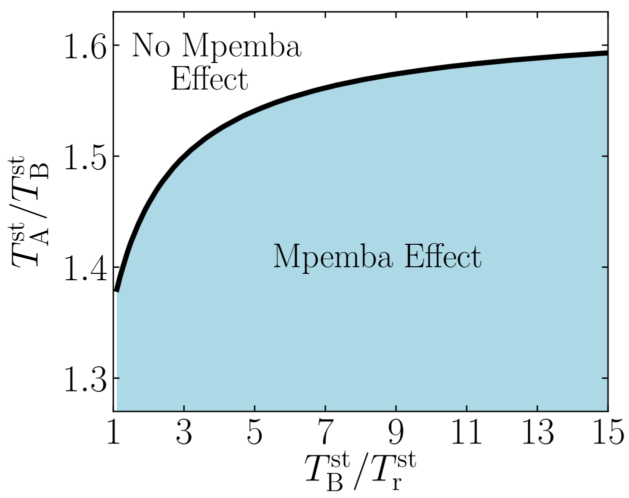

Fig. 1. Phase diagram of the Mpemba effect for $\alpha = 0.9$, $\beta = 0$, and $\kappa = \frac{1}{2}$ resulting from the protocol described in the text.

This leads us to define a protocol for the initialization of a pair of identical samples of a dilute granular gas of inelastic and rough hard disks at arbitrary initial states, from which Mpemba effect is expected to occur for a thermal reservoir with ε_{r} and $T_{\mathrm{r}}^{\mathrm{n}}$, such that the steady state is given by $(\theta_{\mathrm{r}}^{\mathrm{st}}, T_{\mathrm{r}}^{\mathrm{st}})$. The protocol can be summarized as follows: one sample, say A, is put in contact with a thermostat with $\varepsilon_{\mathrm{A}} = 0$ and $T_{\mathrm{A}}^{\mathrm{n}} > T_{\mathrm{r}}^{\mathrm{n}}$, whereas the other system, B, is put in contact with a bath characterized by $\varepsilon_{\mathrm{B}} = 1$ and $T_{\mathrm{A}}^{\mathrm{n}} > T_{\mathrm{B}}^{\mathrm{n}} > T_{\mathrm{r}}^{\mathrm{n}}$, so that $\theta_{\mathrm{B}}^{\mathrm{st}} > \theta_{\mathrm{r}}^{\mathrm{st}} > \theta_{\mathrm{A}}^{\mathrm{st}}$ and $T_{\mathrm{A}}^{\mathrm{st}} > T_{\mathrm{B}}^{\mathrm{st}} > T_{\mathrm{r}}^{\mathrm{st}}$ once the systems are thermalized. These steady-state pairs of values, $(\theta_{\mathrm{A}}^{\mathrm{st}}, T_{\mathrm{A}}^{\mathrm{st}})$ and $(\theta_{\mathrm{A}}^{\mathrm{st}}, T_{\mathrm{B}}^{\mathrm{st}})$, define a set of *initial* states candidate for observing Mpemba effect when the reservoirs of both samples are changed to have *common* $T_{\mathrm{r}}^{\mathrm{n}}$ and ε_{r}. The latter parameter is conveniently chosen as

$$\varepsilon_{\mathrm{r}} = \frac{1-\beta+2\kappa}{1-\beta+2\kappa+\kappa(1+\beta)}, \tag{3}$$

so that, according to our theoretical description, $\theta_{\mathrm{r}}^{\mathrm{st}} = \frac{1}{2}\left(\theta_{\mathrm{A}}^{\mathrm{st}} + \theta_{\mathrm{B}}^{\mathrm{st}}\right)$.

Figure 1 shows the phase diagram in the plane $T_{\mathrm{A}}^{\mathrm{st}}/T_{\mathrm{B}}^{\mathrm{st}}$ vs $T_{\mathrm{B}}^{\mathrm{st}}/T_{\mathrm{r}}^{\mathrm{st}}$, indicating the region where Mpemba effect is present for the case $\alpha = 0.9$, $\beta = 0$, and $\kappa = \frac{1}{2}$.

We will validate such theoretical description via direct simulation Monte Carlo method and event-driven molecular dynamics simulations.

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