Bifurcation structure of traveling pulses in Type-I excitable media

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Excitability is a property of certain nonlinear dynamical systems concerning their response to external perturbations. Excitable systems can be classified into two classes, Type-I and II, with differentiated dynamical properties and obtained through different bifurcations [1]. Excitable media, locally excitable spatial extended systems, show different regimes in which local perturbation, exceeding a threshold, can propagate across the medium. There are many studies considering Type-II excitable media, but much less is known about pulse propagation in the Type-I case. Recently, several vegetation systems compatible with Type-I excitability have shown traveling pulses [2, 3, 4], renewing interest in their study.

We have studied the existence of traveling pulses in a general Type-I excitable 1-dimensional media. We have obtained the stability region and characterized the different bifurcations behind either the destruction or loss of stability of the pulses. In particular, some of the bifurcations delimiting the stability region have been connected, using singular limits, with the two different scenarios that mediated the Type-I local excitability, i.e. homoclinic (saddle-loop) and SNIC (Saddle-Node on the Invariant Circle) bifurcations. This connection explain the similarities between the pulses profile and excitable trajectories pointed out in [5]. Finally, the existence of these pulses has been tracked, outside the stability region, to a drift pitchfork instability of localized steady structures.

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