Long-lived non-equilibrium state in a molecular fluid with non-linear drag

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Glassy behaviour is typically associated with systems with many distinctly interacting units, which give rise to a complex energy landscape with multiple minima separated by barriers. The typical phenomenology of glassy systems includes, among other aspects, strongly non-exponential relaxation. Here, we study the dynamical evolution of a fluid with non-linear drag, for which binary collisions are elastic, described at the kinetic level by the Enskog-Fokker-Planck (EFP) equation

$$\partial_t f = \frac{\partial}{\partial \mathbf{v}} \cdot \left[\zeta(v) \left(\mathbf{v} + \frac{k_B T_s}{m} \frac{\partial}{\partial \mathbf{v}} \right) f \right] + J[\mathbf{v}|f,f]. \quad (1)$$

In the above, $f = f(\mathbf{v}, t)$ is the one-particle velocity distribution function (VDF), k_B the Boltzmann's constant, T_s the bath temperature, m the mass of the fluid particles, $J[\mathbf{v}|f,f]$ the Enskog collision operator, and

$$\zeta(v) = \zeta_0 \left(1 + \gamma \frac{m v^2}{k_B T_s} \right) \quad (2)$$

the non-linear drag coefficient [1]. In the latter, γ measures the degree of non-linearity of the drag force.

The dynamical state of the system is determined via the physical variables $(T, \{a_l\}_{l=2}^{\infty})$, where $T(t)$ is the kinetic temperature, and $a_l(t)$ are the so-called Sonine coefficients, which measure deviations from the local equilibrium Maxwellian distribution

$$f_{\text{LE}}(\mathbf{v}, T(t)) = \left(\frac{m}{2 k_B T(t)} \right)^{\frac{d}{2}} e^{-\frac{m v^2}{2 k_B T(t)}}, \quad (3)$$

i.e. $a_l(t) = 0$, for all l, if $f = f_{\text{LE}}$.

We observe that a non-exponential relaxation function for the kinetic temperature appears when the system is quenched to low enough temperatures, such that $\theta_i \equiv T_i/T_s \gg 1$, with θ_i being the ratio between the initial and final temperatures. This relaxation is universal in the sense that, after a suitable rescaling of the variables, it does not depend on the initial and final temperatures, nor on the degree of non-linearity, nor on the relevance of the collision term. This phenomenon is related to the existence of a long-lived non-equilibrium state (LLNES), in which the Sonine coefficients approach large stationary values a_l^r. In such state, the temperature decays algebraically, specifically

$$T(s) = \frac{T_i}{1 + 2(d+2)(1 + a_2^r) s}, \quad (4)$$

with $s \equiv \gamma \theta_i t$ being a new timescale. Figure 1 shows the comparison between our theoretical prediction for the relaxation function and different sets of data obtained through *Direct Simulation Monte Carlo* (DSMC), which allows to solve the EFP equation numerically. As one may observe, the agreement between simulations and theory is excellent.

The physical picture behind the LLNES is appealing. The limit $\theta_i \gg 1$ is roughly equivalent to setting $T_s \to 0$, which removes the diffusion, stochastic term from the EFP equation. Thus, in the absence of collisions, the system behaviour is purely governed by the non-linear drag, and the dynamics of the particles become deterministic. In fact, we show that the VDF at the LLNES becomes a Dirac delta function in velocity space,

$$f_{\text{LLNES}}(\mathbf{v}, t) \propto \delta \left(v^2 - \frac{d k_B T(t)}{m} \right), \quad (5)$$

where we may identify $v_T(t) \equiv \sqrt{2 k_B T(t)/m}$ as the thermal velocity. Thus, in this regime, the motion of the particles is completely determined by the kinetic temperature.

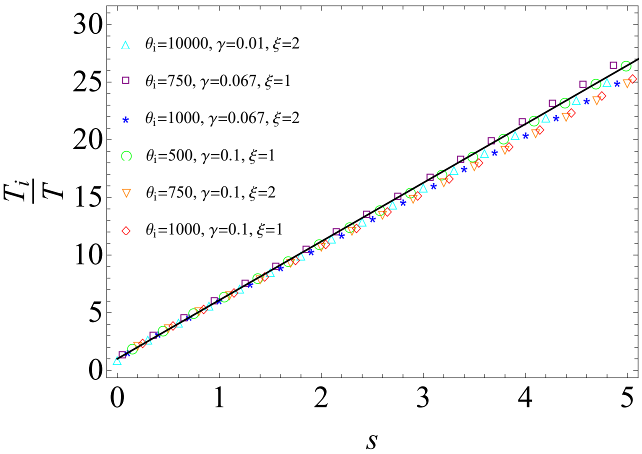

Fig. 1. Relaxation after a quench to a low temperature. Specifically, we plot T_i/T as a function of the scaled time $s = \gamma \theta_i t$. Data from DSMC correspond to parameters (θ_i, γ , ξ), as specified in the legend, and $d = 2$. In the above, ξ stands for the inverse collision rate ($\xi \to +\infty$ corresponds to the collisionless case). The linear behaviour in T_i/T implies that the temperature relaxes algebraically, basically as t^{-1}, as theoretically predicted by Eq. (4) (solid line).

[1] A. Patrón, B. Sánchez-Rey, and A. Prados, *Strong non-exponential relaxation and memory effects in a fluid with non-linear drag*, Phys. Rev. E **104**, 064127 (2021).