Time crystal oscillations in a laser system with weak periodic forcing

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In many-particle systems, when the translation symmetry in space (in time) is spontaneously broken, the result is a space (time) crystal. Time-crystal states are characterized by highly regular oscillations that are stable over very long times, are robust under perturbations ("rigidity") and break time-translation symmetry [1]. In systems under periodic forcing, the forcing defines the discrete time translation symmetry and this symmetry is broken in the time crystal state, where the system's variable displays sub-harmonic oscillations that are rigidly locked to the driving signal.

A key requirement for observing time-crystal oscillations is that they have long-term regularity that results from the interaction of many degrees of freedom. This excludes period-doubling oscillations in low-dimensional systems, and it also excludes oscillations in mode-locked lasers, which arise from the interactions of many modes but which lack long-term regularity due to noise.

Here we address the following question: Can a time-delayed feedback loop counteract the effect of noise and generate long-term order?

Time-delayed systems (TDSs) with feedback loops, governed by equations of the form $du(t)/dt = f(u(t), t) + Ku(t - \tau)$, have an infinite phase space because the initial condition is the function $u(t)$ defined in [-τ,0], where τ is the feedback delay time. In this type of TDS, when τ is long, analogies have been found with the dynamics of one-dimensional spatially extended systems (1D SESs). Specifically, in the TDS, τ plays the role of the size, L, in a 1D SES. When τ is long enough, using the so-called space-time representation, complex spatio-temporal behavior has been found (such as pattern formation, defects and localized structures), analogous to those occurring in 1D SESs.

These analogies make periodically driven stochastic TDSs promising test benches for finding time-crystal oscillations. A semiconductor laser with optical feedback is a well-known TDS, in which feedback from a distant reflector generates a complex dynamics that, when viewed using the space-time representation, reveals spatio-temporal structures typical of 1D SES.

Near threshold and for appropriated feedback conditions the laser output intensity displays spikes that occur at irregular times (Fig. 1a). We have found [2] that a weak periodic modulation of the laser current can lock the spikes to the modulation. With a pulsed waveform we found, depending on the modulation frequency, harmonic or subharmonic locking (i.e., there is a spike every one or more modulation cycles, Fig. 1b); in contrast, with sinusoidal modulation there is no harmonic locking (Fig. 1c) but sub-harmonic locking (for a higher modulation frequency a spike is emitted every two cycles, Fig. 1d). In the locked states the spikes become periodic, but in between the spikes, the intensity fluctuations are still irregular.

When calculating the Fano factor (F, a well-known measure of the variability of a sequence of events), for particular locking conditions we found [3] long-range regularity in the timing of the spikes revealed by very small F values (Fig. 2). Therefore, we conclude that we have found an experimental stochastic high-dimensional laser system where subharmonic oscillations with time-crystal characteristics are generated by sinusoidal modulation and delayed feedback.

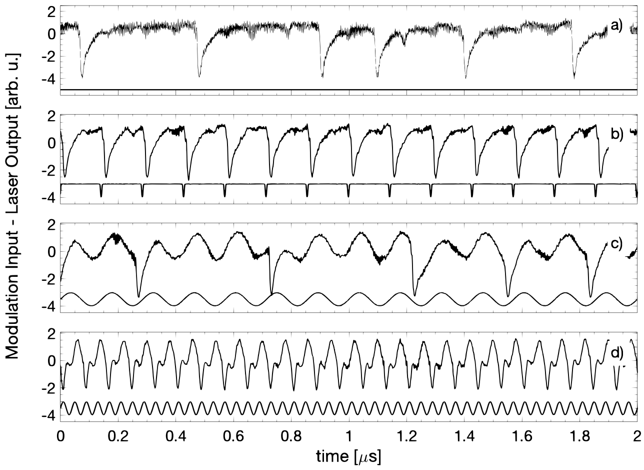

Fig. 1. Laser intensity without modulation (a) and with pulsed modulation (b) with $f_{mod} = 7$ MHz. (c), (d) laser intensity with sinusoidal modulation with $f_{mod} = 7$ MHz and 25 MHz respectively.

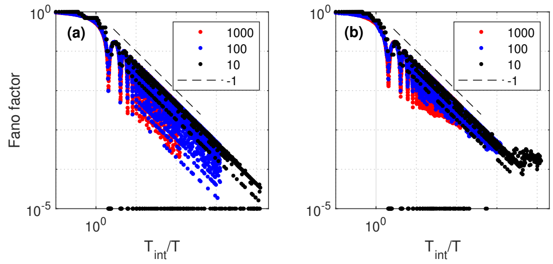

Fig. 2. Fano factor of the sequence of spikes generated by sinusoidal modulation with $f_{mod} = 25$ MHz. In (a) F is calculated from the original spike sequence and in (b) from the shuffled one, using 10, 100 or 1000 spike counting intervals. To represent $F = 0$ in log scale, we set it to $F = 10^{-5}$.

[1] N. Y. Yao and C. Nayak, *Time crystals in periodically driven systems*, Physics Today **71**, 9, 40 (2018).

[2] J. Tiana-Alsina *et al.*, Experimental study of modulation waveforms for entraining the spikes emitted by a semiconductor laser with optical feedback. *Opt. Express* **26**, 9298 (2018).

[3] J. Tiana-Alsina and C. Masoller, Dynamics of a semiconductor laser with feedback and modulation: experiments and model comparison. *Opt. Express* in press (2022).