Noisy Voter Model with time-varying influencers

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Abstract

The noisy voter model (NVM) [1] describes emergent behavior of interacting agents choosing between two or more opinions. The adjective 'noisy' indicates that an agent can change their choice by imitating that of a neighbor. When the imitation mechanism is stronger than the noise, the stationary distribution of the fraction of agents choosing one opinion is bimodal. On the other hand, if the noise is stronger the resulting stationary distribution is unimodal.
Several variations of the NVM have been proposed; in [2], for example, the authors study how the presence of agents that never change opinion, so-called *Zealots*, affects the emergence of consensus.
In this work we enrich the analysis of external influence on the NVM by introducing a group of agents that change opinion only at random, but who are not subject to the herding process. We call these '*Influencers*'. This is motivated by *Influence marketing*, that is when a brand enrolls influencial people in order to change the buying habits of consumers. Nevertheless, a group of influencers may switch the endorsed brand. The goal of this work is to model and analyze this phenomenon and understand if the switch in the influencers may prevent consensus on a product.

If we use the notation X_A for a normal voter that chooses A (and similarly, X_B) and Y_A (Y_B) for an influencer that chooses A (B), the dynamics of the model results in the following reactions:

$$\begin{aligned} X_A + X_B &\xrightarrow{h} X_B + X_B \\ X_A + Y_B &\xrightarrow{h} X_B + Y_B, \\ X_B &\xrightarrow{a} X_A, \\ X_A &\xrightarrow{a} X_B \end{aligned} \quad (1)$$

Additionally, the influencers change between states A and B with rate λ. This can occur collectively (i.e., all influencers must change at once), or they can change individually or in multiple groups. For the purposes of the population of normal agents, the state of the influencers sets an 'environmental state', σ.

Assuming that there are N normal agents and that n of these are in state A, the system can be described by a master equation of the type

$$\partial_t P(n,\sigma,t) = \mathbf{M}_\sigma P(n,\sigma,t) + \lambda \mathbf{A} P(n,\sigma,t), \quad (2)$$

where the operator $\mathbf{M}_\sigma$ describes the evolution of normal agents in environment σ, and $\mathbf{A}$ effects the changes of the environmental state. The parameter λ describes the time scale separation between the dynamics of normal agents and that of the influencers.

We now assume that there are αN influencers, who change opinion collectively. The environment then has two states. In the adiabatic limit $\lambda \to \infty$ (infinitely fast switching influencers), the system reduces to a NVM with effective size $\tilde{N} = N/(1+\alpha)$, and noise constant $\tilde{a} = a(1+\alpha) + \frac{\alpha}{2}$.

In the limit of a large, but finite population ($N \gg 1$) we can use methods from [3] to reduce Eq. (2) to

$$\partial_t P(x,\sigma,t) = \mathbf{L}_\sigma P(x,\sigma,t) + \lambda \mathbf{A} P(x,\sigma,t), \quad (3)$$

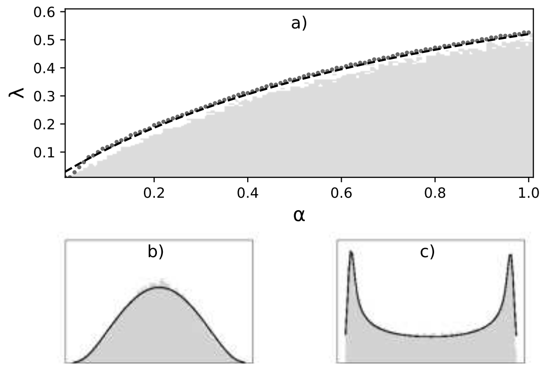

Fig. 1. **a)** is the phase diagram of the NVM with influencers. The dashed line represents the line separating the unimodal and bimodal regimes obtained from the analytical solution for the stationary distribution in the PDMP limit. The dots take into account leading order corrections in $1/N$ in the dynamics. The shaded area is from simulations of the original system. ($N = 100$, $a = 0.01$, $h = 1$).
b) and **c)** are respectively a unimodal ($\lambda = 0.5$ and $\alpha = 0.2$) and a bimodal ($\lambda = 0.2$ and $\alpha = 0.7$) stationary distribution for $N = 200$, $a = 0.01$ and $h = 1$; here the black solid line is the numerical solution that takes into account Gaussian fluctuations.

where $x = \frac{n}{N}$. The operator $\mathbf{L}_\sigma$ is now of the type as in a Fokker-Planck equation, and captures leading and subleading terms in $1/N$. Upon sending $N \to \infty$ this reduces further to a Liouville operator, resulting in a piecewise deterministic Markov process (PDMP).

From these equations the shape of the stationary distribution can be obtained analytically in the PDMP limit, and numerically when Gaussian fluctuations are taken into account. Results are shown in Fig. 1. For a fixed value of α, we observe a phase transition from a bimodal to a unimodal shape of the stationary distribution at a critical value $\lambda_c(\alpha)$. As can be seen from Fig. 1, $\lambda_c(\alpha)$ increases with α, i.e. if there are more influencers (or analogously if their influence is stronger), they need to switch opinion faster in order to prevent the reach of consensus. Indeed, the more influencers there are the faster normal consumers tend to allign with their opinion.

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