Mpemba effect in driven gases: Are the thermal and entropic criteria equivalent?

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The response to an excitation in a complex condensed matter system may depend on its entire history and not only on the instantaneous value of the macroscopic state variables, thus signaling the breakdown of a purely macroscopic description due to memory effects.

The so-called Mpemba effect [1] is a counterintuitive memory phenomenon according to which, given two samples of a fluid, the initially hotter one may cool more rapidly than the initially cooler one. Although initially reported in the case of water [2, 3], its existence for that liquid is still questioned [4].

In this talk, the system chosen to reveal the feasibility of the Mpemba effect is a gas of elastic hard spheres coupled to a fluid reservoir via a nonlinear drag force plus a white-noise stochastic force [5]. The associated (spatially uniform) Enskog–Fokker–Planck equation for the one-body velocity distribution function $f(\mathbf{v},t)$ is

$$\partial_t f(\mathbf{v},t)-\partial_{\mathbf{v}}\cdot\left\{\zeta(v)\left[\mathbf{v}+\frac{k_BT_b}{m}\partial_{\mathbf{v}}\right]\right\}f(\mathbf{v},t)=J[\mathbf{v}|f,f],$$

where $J[\mathbf{v}|f,f]$ is the Boltzmann–Enskog collision operator, $\zeta(v)=\zeta_0\left(1+\gamma mv^2/k_BT_b\right)$ is the velocity-dependent drag coefficient (γ measuring the degree of nonlinearity), and T_b is the temperature of the background fluid.

The study is addressed by minimal descriptions based on kinetic theory (basic and extended Sonine approximations), the theoretical predictions being numerically confirmed by the direct simulation Monte Carlo (DSMC) method and by event-driven molecular dynamics (EDMD) [6].

A direct test of the Mpemba effect consists of checking whether or not the (kinetic) temperature $T(t)=\frac{m}{3}\langle v^2\rangle$ of two independent samples A and B cross each other; this defines the *thermal* criterion. An alternative *entropic* criterion, on the other hand, is based on the possible crossover of the relative entropy $\mathcal{D}(t)=\langle\ln\left[f(\mathbf{v},t)/f^{\text{eq}}(\mathbf{v})\right]\rangle$, where $f^{\text{eq}}(\mathbf{v})$ is the equilibrium distribution.

As will be discussed in the talk, if the Mpemba effect takes place during the kinetic stage of the relaxation to equilibrium, it is possible to find the thermal effect but not the entropic one (see Fig. 1 for an illustration), or vice versa (see Fig. 2 for an illustration). As observed in Figs. 1 and 2, an excellent agreement between simulation data and the minimal kinetic-theory description exists.

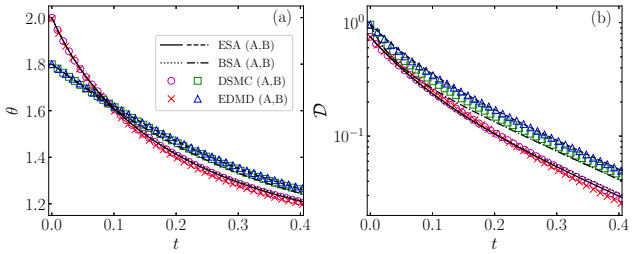

Fig. 1. Relaxation of (a) the scaled temperature $\theta(t)\equiv T(t)/T_b$ and (b) the relative entropy $\mathcal{D}(t)$ for two samples A and B initially prepared with $(\theta_A^0,\theta_B^0)=(2,1.8)$ and $(a_{2A}^0,a_{2B}^0)=(0.5,-0.35)$, where a_2 represents the excess kurtosis. Here, $\zeta_0=1$, $\gamma=0.1$, and the unit of time is the equilibrium mean free time.

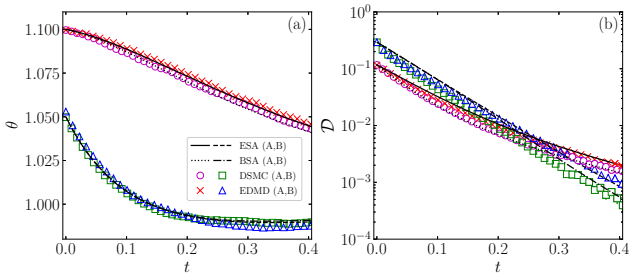

Fig. 2. Same as in Fig. 1, except that the initial conditions are $(\theta_A^0,\theta_B^0)=(1.1,1.05)$ and $(a_{2A}^0,a_{2B}^0)=(-0.35,0.5)$.

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