Numerical simulations on the Potts model using algorithms based on transition rate

Lourdes Bibiana Merino-Solís[1], and Francisco Sastre[1]
[1]División de Ciencias e Ingenieras, Campus León de la Universidad de Guanajuato,
Loma del Bosque 103, Lomas del Campestre CP 37150, León, Gto., México

In this work we implement a new algorithm based on transitions rates between order parameter levels in the two dimensional q-state Potts model without external field.

The Potts model has been widely studied and is one of the few interacting models with analytical solution, in the two dimensional case without external field [1]. Despite its simplicity it can be used to study several systems such as magnetic dominions grow [2, 3, 4], cellular behavior [5, 6] or social and demographic behavior [7, 8], for name a few.

However, the study of the phase diagram when external fields are included, has been proved to be very difficult to establish using the standard numerical methods. This is the main reason to use a new numerical method to study the Potts model, as a first step without external field.

The algorithm is based on the determination of the transition rates of between discrete macroscopic observables. This numerical method can to evaluate directly the derivative of the logarithm of the probability distribution function with respect to the order parameter [9]. Using standard finite-size scaling theory we evaluate the critical point and the correlation length critical exponent for q = 2, 3 and 4.

Our results are in good agreement with the expected theoretical value [1]. Theoretical values are as follows: $T_c = 0.9949$, $\nu = 5/6$ for $q = 3$ and $T_c = 0.9102$, $\nu = 2/3$ for $q = 4$, while the results obtained were $T_c = 0.9896 \pm 0.0075$, $\nu = 0.7735 \pm 0.0628$ for $q = 3$ and $T_c = 0.9087 \pm 0.0085$, $\nu = 0.6425 \pm 0.0458$ for $q = 4$.

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