Ordering Dynamics and Path to Consensus in Multi-State Voter Models

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In this work, we study the Voter Model with a general number M of opinion states, the multi-state voter model (MSVM)[1, 2], with a focus in the coarsening process. The system consists of N interacting agents placed on the nodes of a network. Agents can be in one of the M possible opinion states ($M \leq N$) and they interact with each other through an imitation process as follows: (i) a random voter is chosen; (ii) the selected voter takes the opinion of a randomly chosen neighbor; (iii) repeat until consensus is reached. We analyze the time evolution of the dynamics and the approach to consensus when the agents are placed in the nodes of a Complete Graph (CG) or in the nodes of uncorrelated networks such as the Erds Renyi (ER) or the Barabsi-Albert (BA) network. The approach to consensus is studied through the time evolution of the density of interfaces ρ, that is the fraction of links that connects the agents with different opinions.

The time evolution of the density of interfaces averaged over realizations, $< \rho >$, was analytically studied through rate equations for the MSVM on the CG and by means of the pair approximation [3, 4] for the uncorrelated networks. In both cases, $< \rho >$ has an exponential decay with a survival time τ that depends on the system size and not on the number of opinion states. It stands out that the absorbing state is only reached for finite size systems, while in the thermodynamic limit, the system stays in a metastable state. The analytical results were confirmed by numerical simulations.

The log-log plot of $< \rho >$ shows that the system stays in a plateau for a finite time before going to the absorbing state. The value of the plateaux, ξ, depends on M and on the mean degree $\overline{k}$ in the case of the ER and BA(Fig 1). The plateau value has a growing behavior with M.

Individual realizations of the MSVM show that finite systems initially fluctuates around a plateau and then ρ falls to successive intermediate plateaux as opinion states disappear, before going to the absorbing state. This behavior is not observed when the ensemble average is considered, so we performed restricted averages over realisations that only have M^* opinion states left to found intermediate plateaux (Fig 2).

After an extinction, the probability distribution of the fraction of agents in the surviving opinions states, $P(x_1, ..., x_{M^*})$, is found to be flat (this is, all $(x_1, ..., x_{M^*})$ such that $x_1 + ... + x_{M^*} = 1$ are equally likely) and the intermediate plateau value can be analytically obtained from the marginal distribution for a given M^*.

Finally, we introduced zealots (agents that do not change their opinion state during the dynamics) in order to engineer a MSVM that exhibits stable states equivalent to those intermediate plateaux of mixed opinions that were observed in the average over restricted realizations. By means of numerical simulations, we found that the addition of Z competing zealots creates stable states and, when there is one zealot of each opinion state, ρ stays in a steady state located at the same value than the intermediate plateau when only $M^* = Z$ opinion states are left (Fig 2). The correspondence between both plateaux is due to the fact that in the stationary state, when a zealot per opinion is introduced, the probability distribution of fraction of agents in each opinion is also flat.

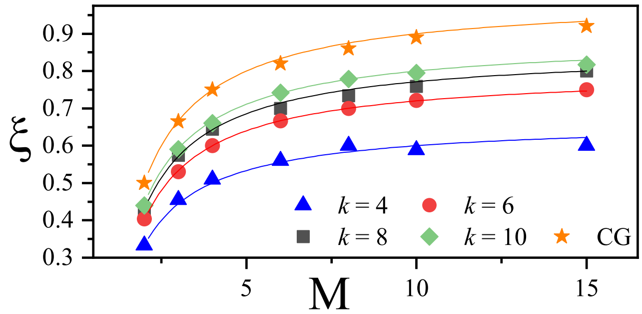

Fig. 1. Value of the plateau level ξ as function of M for the CG and networks with different mean degree $\overline{k}$. The lines are the analytical prediction. Initially, the opinions are equally distributed among the agents.

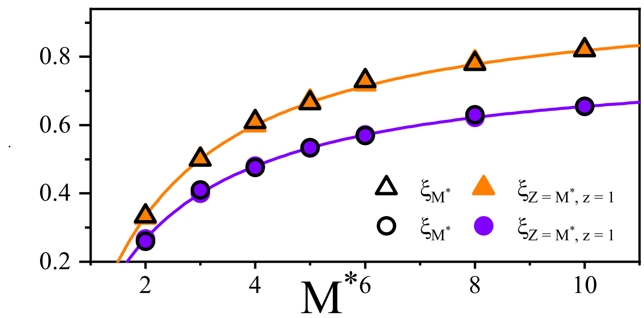

Fig. 2. Open triangles (circles) correspond to the plateau value for a MSVM on a CG (ER) when, after successive extinctions, there are M^* opinion left; full triangles (circles) correspond to the steady-state for a MSVM on a CG (ER) with a single zealot, $z_i = 1$, in each of the M^* possible opinion states. Both values are in good agreement.

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