Aging effects in complex contagion

David Abella-Bujalance[1], Maxi San Miguel[1] and Jose Javier Ramasco[1]
[1] Instituto de Fsica Interdisciplinar y Sistemas Complejos IFISC (CSIC - UIB), Campus UIB, 07122 Palma de Mallorca, Spain

We study the effects of heterogeneous timing interactions in processes of complex contagion, focusing on the threshold model [1] with exogenous and endogenous aging. For the threshold model, the binary state variable indicates if either the agent has adopted a technology or not. Endogenous aging is considered as the property of agents in the system to be less prone to change state the longer they have been in the current state. On the other hand, in exogenous aging, memory is lost after failed attempts to change state [2].

In both cases, numerical simulations show that aging slows the cascade dynamics towards the fully adopted state. The exponential increase of the fraction of adopted agents $\rho(t)$ exhibited by the original threshold model is replaced by a stretched exponential or power-law increase when aging mechanism is exogenous or endogenous, respectively (see Fig.1). This behaviour is universal for different system sizes, networks and values of the control parameters (average degree z and threshold T).

The memory dependent dynamics induced by aging cannot be treated with standard methods for binary-state dynamics in networks [3]. We derive an approximate master equation (AME) reducing the non-Markovian dynamics to Markovian by enlarging the number of variables [4]. Our AME describe binary state dynamics with timing interactions for any network generated with the configuration model. For the threshold model with aging, the numerically integrated solutions give a good agreement with numerical simulations (Fig. 1).

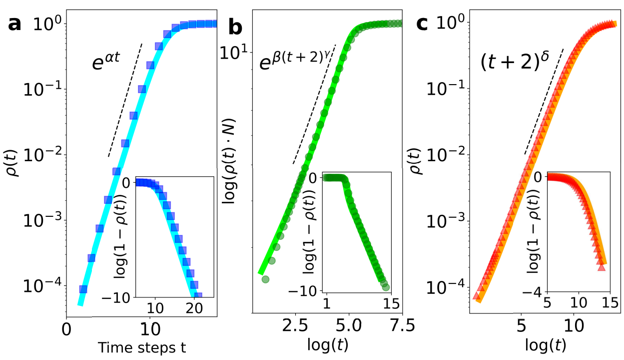

Fig. 1. Cascade dynamics and fall to the adopted state $\rho = 1$ of the Watts threshold model (a) and the versions with exogenous (b) and endogenous (c) aging effects. The underlying network is a 3-regular random graph and the homogeneous threshold is $T = 0.2$. The exponent values are $\alpha \simeq 1.0$, $\beta \simeq 1.14$, $\gamma \simeq 0.38$ and $\delta \simeq 1.0$. AME solutions (solid lines) describe accurately the numerical results.

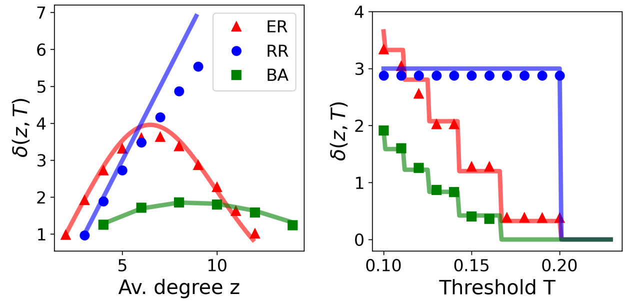

Fig. 2. Exponent δ dependence on the average degree z ($T = 0.1$) (left) and the T (right) for the threshold model with endogenous aging. Different markers indicate results from numerical simulations with different topology: red triangles indicates an Erds-Renyi (ER), blue circles indicate a Random Regular (RR) and green squares indicate a Barabasi-Albert (BA) graphs. In (b), average degree is fixed $z = 5$ for ER and RR, and $z = 8$ for the BA. Predicted values by Eq.1 (solid lines) fit the results for each topology. System size is fixed at $N = 160000$.

We reduce the AME for the threshold model with aging to a set of two coupled differential equations. The equations are linearized to find an analytical solution of the fraction of adopted agents $\rho(t)$. The exponential increase for the original model and the power law dynamics for the version with endogenous aging are predicted. In fact, the exponent is found to coincide:

$$\delta(Z,T) = \alpha(z,T) = \sum_{k=0}^{1/T} \frac{k(k-1)}{z} p_k - 1. \qquad (1)$$

This exponent dependence is shown to be different according to the degree distribution of the underlying network p_k. Values computed from numerical simulations are in good agreement with analytical predictions (see Fig.2).

[1] Watts, D. J. *A simple model of global cascades on ran- dom networks,* Proceedings of the National Academy of Sciences **99**, 5766 (2002).).

[2] J. Fernndez-Gracia, V. M. Eguiluz, and M. San Miguel, *Update rules and interevent time distributions: Slow or- dering versus no ordering in the voter model,* Physical Review E **84**, 10.1103/physreve.84.015103 (2011).

[3] Gleeson, J. P. *Binary-State Dynamics on Complex Networks: Pair Approximation and Beyond.*, Physical Review X, **3**(2), (2013).

[4] Peralta, A. F., Khalil, N., and Toral, R. *Reduction from non-Markovian to Markovian dynamics: the case of aging in the noisy-voter model.* Journal of Statistical Mechanics: Theory and Experiment, **2** 024004 (2020)