Survival and extreme statistics of work, heat, and entropy production in steady-state heat engines

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In small systems, fluctuations play a prominent role, often pushing systems far away from equilibrium. As a consequence, they are of crucial importance for the performance and robustness of microscopic motors, heat engines and refrigerators, where the work extracted, the heat dissipated, and the efficiency over a finite time, become stochastic quantities that can be described within the framework of stochastic thermodynamics. However, it remains an open, active area of research to derive universal "survival" statistics (e.g. the probability to remain below or above a given threshold) of fluctuating physical quantities in steady-state nonequilibrium processes and in particular those performing some useful thermodynamic task. More precisely, it is relevant to study extreme and survival statistics of the work extracted by microscopic heat engines, of the heat dissipated into the environment, or the peaks in the consumption of chemical fuel driving a molecular machine, since they can shed new light on the function and properties of such systems. In this context, some important questions are: (i) are there universal bounds on the statistics of entropy production maxima and minima during a prescribed interval? (ii) what is the survival probability for the work or heat not to exceed or fall below a certain threshold value? (iii) what is the "optimal" threshold that guarantees a prescribed value of the survival probability for the work extracted by a stationary heat engine?

In this talk, based on reference [1], I will discuss some novel insights about the above questions. First, I will show how to derive universal inequalities for the cumulative distribution of the finite-time maximum and minimum of stochastic entropy production and their averages in generic nonequilibrium stationary states. These nonequilibrium relations entail a new development of martingale theory for entropy production [2, 3, 4] and substantially extend and generalize previous results on entropy production minima statistics [4]. Then I will show how to apply these results to bound the survival statistics of the work extracted and the heat dissipated by steady-state engines permanently coupled to two heat baths at cold T_c and hot T_h temperatures. Interestingly, we obtain time-dependent thresholds $w_\pm(\tau)$ that allow us to bound the extreme fluctuations of the stochastic work and heat with a given confidence level, for a time interval $[0, \tau]$. The main results are tested with numerical simulations of a stochastic photoelectric device, as illustrated in Fig. 1.

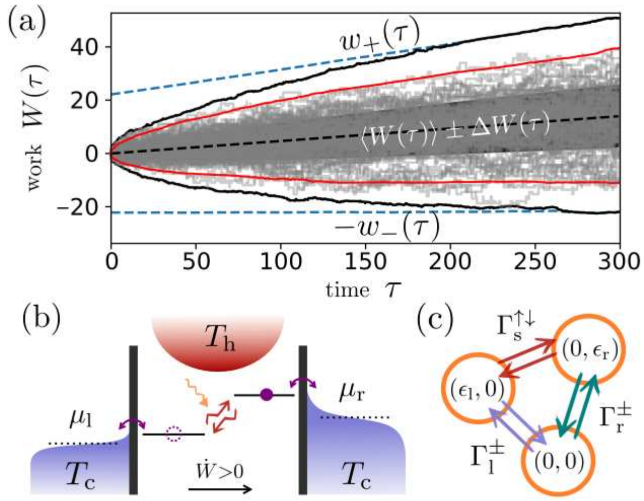

Fig. 1. **(a)** Illustration of work extreme fluctuations and the optimal threshold bounds $w_\pm(\tau)$ for its maxima (upper black and red solid lines) and minima (bottom black and red solid lines) developed by a photoelectric device during the interval $[0, \tau]$ as a function of τ. Gray thin lines represent the stochastic work extracted (in $k_\mathrm{B}T$ units) for a sample of 100 trajectories. The solid lines are optimal thresholds $w_+(\tau)$ (top) and $w_-(\tau)$ (bottom) for a confidence value of 99% ($\alpha = 0.01$, black lines), and 90% ($\alpha = 0.1$, red lines). The average work output $\langle W(\tau)\rangle$ (black dashed line) and its standard deviation $\Delta W(\tau)$ (dark shadow area) are also shown for comparison. The blue dashed lines are linear optimal asymptotic thresholds for $\alpha = 0.01$. **(b)** Photoelectric device composed by two (single-level) quantum dots transporting electrons between their respective fermionic reservoirs at temperature T_c against a chemical potential difference ($\mu_\mathrm{r} \geq \mu_\mathrm{l}$) powered by photons at temperature T_h from a hot source. **(c)** Energetic states of the device and relevant rates producing transitions between them (simultaneous occupancy of the two dots is avoided by Coulomb repulsion).

[1] G. Manzano, and É. Roldán, *Survival and extreme statistics of work, heat, and entropy production in steady-state heat engines*, arXiv: 2109.03260 (2021) — accepted in Phys. Rev. E.

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[4] I. Neri, É. Roldán, and F. Jülicher, *Statistics of Infima and Stopping Times of Entropy Production and Applications to Active Molecular Processes*, Phys. Rev. X **7**, 011019 (2017).