Random walkers on deformable media

Carlos Lajusticia Costán[1], Silvia N. Santalla[2], Javier Rodríguez-Laguna[1], and Elka Korutcheva[1]
[1]Universidad Nacional de Educación a Distancia (UNED), Madrid, Spain
[2]Dpto. Física & GISC, Universidad Carlos III de Madrid, Spain

We consider random walkers that deform the medium as they move, enabling a faster motion in regions which have been recently visited. This induces an effective interaction between walkers mediated by the medium, which can be regarded as a space metric. Such an effect gives rise to a statistical mechanics toy model for gravity, motion through deformable matter or adaptable geometry. In the strong-deformability regime, we find that diffusion is ruled by the *porous medium equation*, thus yielding subdiffusive behavior of an initially localized cloud of particles, whose global width will grow like $\sigma \sim t^{1/3}$, though the sample-to-sample width average will sustain a $\sigma \sim t^{1/4}$ or $t^{1/2}$ growth. Indeed, random walkers present anti-persistence and strong memory effects, which we explore indirectly through the fluctuations of the center of mass of the cloud.

Our model of *random walkers on a deformable metric* (RWDM), is built from of N_p random walkers on a chain. Any particle standing on site i has a certain probability per unit time $J_{i,i-1}$ of hopping to site $i-1$, and $J_{i,i+1}$ of hopping to site $i+1$. We will assume these probabilities are symmetric and the system starts with uniform hopping rates, $J_i = J_0$ for all i. Furthermore, whenever a particle jumps across a link $(i, i+1)$ the hopping probability is updated, $J_i \to J_1 \geq J_0$. If no walker crosses the link during the subsequent time-steps, its hopping probability will decay towards the relaxed value J_0 in a time of order t_0. Figure 1 provides a graphical description of our model.

On the other hand, a theoretical approach to the problem will lead us to the following equation for the probability distribution of the walkers, $P(x,t)$,

$$\partial_t P(x,t) = b(\partial_x P(x,t))^2 + (a + bP(x,t))\partial_x^2 P(x,t), \quad (1)$$

where $a = J_0$ and $b = J_1 - J_0$. We consider the physical picture provided by Eq. (1) for different values of its parameters. Notice that if $a \to 0$, Eq. (1) corresponds to the well-known *porous medium equation* [1].

In our first model [2] where initally all the walkers were located at the same point, the average over different samples differs substantially from the time-average over a single sample. Intuitively, the reason is that the walkers cloud tends to move more rigidly than expected for non-interacting random walkers. This phenomenon can be characterized by the fluctuations in the position of the center of mass of the cloud.

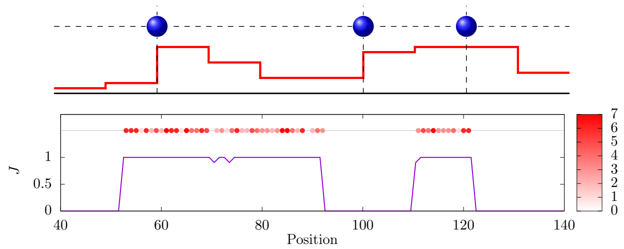

Fig. 1. Top: Illustration of our RWDM model. Walkers occupy integer positions of a chain, and hopping probabilities between neighboring sites can be different. Bottom: A configuration for $J_0 = 10^{-6}$, $J_1 = 1$, $t_0 = 10$ and $N_P = 100$ particles on $L = 200$ sites, after $T = 10^8$ time-steps. The purple line denotes the hopping probabilities, the red balls represents the positions of the particles, and the color intensity denotes the number of particles at a certain site. Notice that most hopping probabilities are near J_0 or J_1, and that the particles are neatly divided into two blocks.

Using a random initial configuration leads to a substantially different evolution of the system, where we can see some clusters start to appear after enough time has been allowed.

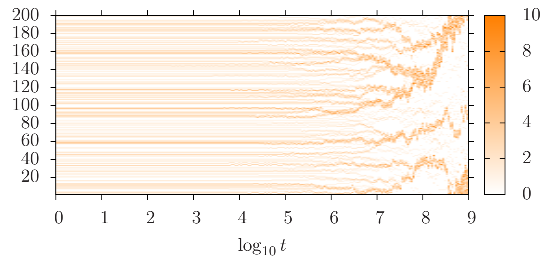

Fig. 2. The illustration shows after $T = 10^9$ time-steps the particules tend to concentrate mostly around two well-defined areas creating two prominent clusters.

[1] Juan Luis Vazquez, *The Porous Medium equation*, Clarendon Press Oxford (2007)

[2] Carlos Lajusticia-Costan et al J. Stat. Mech. (2021) 073207