Analytic exact solution of the one-dimensional triangle-well and ramp fluids

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We present a comprehensive study on the thermophysical and structural properties of two prototypical classes of fluids confined in a one-dimensional line, namely the triangle-well and the ramp potentials. Both potentials are finite-ranged and have an impenetrable core of diameter σ plus a continuous linear part between $r = \sigma$ and $r = \lambda$ (see Fig. 1). While the mathematical form of that additional part is analogous in both cases, the physical meaning is not. In the triangle-well potential the tail is attractive and, apart from its own physical interest, the main importance of the potential resides in representing the effective colloid-colloid interaction in the Asakura–Oosawa mixture. On the other hand, the ramp potential is purely repulsive with a softened core between $r = \sigma$ and $r = \lambda$.

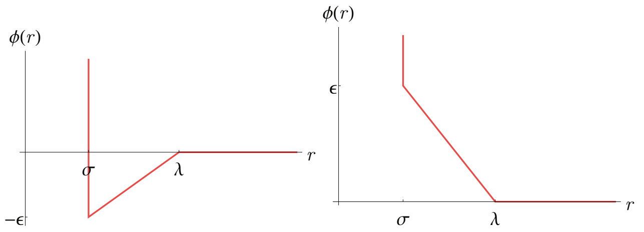

Fig. 1. Sketch of the triangle-well potential (left) and the ramp potential (right).

The exact statistical-mechanical solution in the isothermal-isobaric ensemble for general nearest-neighbor interactions [1] is applied to the study of equilibrium properties of these two classes of fluids, such as the equation of state, the excess internal energy per particle, the structure factor $S(k)$, the direct correlation function $c(r)$, and the radial distribution function $g(r)$. In the latter case, in contrast to previous studies where $g(r)$ was obtained numerically from $S(k)$ by Fourier inversion [2], a fully analytic representation for $g(r)$ is derived in terms of a finite number of coordination-shell terms for any finite r [3]. As an illustration, Fig. 2 shows $g(r)$ at some representative states.

In addition, scatter plots of the bridge function $B(r)$ versus the indirect correlation function $\gamma(r) \equiv g(r) - 1 - c(r)$ are used to gauge the reliability of the hypernetted-chain, Percus–Yevick, and Martynov–Sarkisov closures.

Lastly, the Fisher–Widom line (separating a repulsive-dominated region from an attractive-dominated one) and the Widom line (marking the states with a maximum correlation length at a given temperature) are obtained from the poles of the Laplace transform of $g(r)$ in the case of the triangle-well model. In the ramp potential, being purely repulsive, the decay of the total correlation function is always oscillatory.

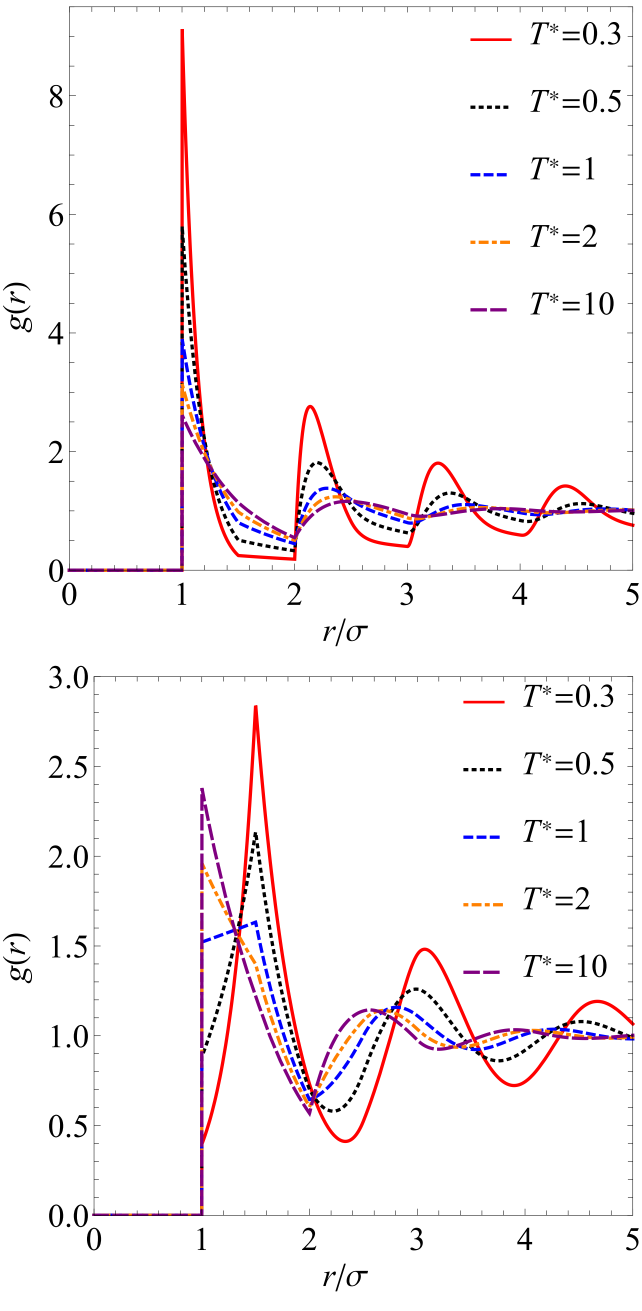

Fig. 2. Radial distribution function $g(r)$ at several representative temperatures for a reduced density $n^* = 0.6$ for the triangle-well potential (top) and the ramp potential (bottom).

[1] A. Santos, *A Concise Course on the Theory of Classical Liquids. Basics and Selected Topics*, Lecture Notes in Physics, vol. 923 (Springer, New York, 2016).

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[3] A. M. Montero and A. Santos, J. Stat. Phys. **175**, 269 (2019).