Dynamic properties in a collisional model of a confined quasi-two-dimensional granular mixture

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Granular mixtures are usually modelled as a mixture of smooth inelastic hard spheres of masses m_i, diameters σ_i, and coefficients of normal restitution α_{ij} $(i,j=1,2,\cdots,s)$. Here, s means the number of components or species of the mixture. Since the total kinetic energy of the mixture decreases in time, in order to maintain the system in rapid flow conditions an external energy input is needed to inject energy into the system and compensate for the energy dissipated by collisions. When both mechanisms cancel each other, the system achieves a steady nonequilibrium state. The injection of energy can be done, for instance, by vibrating walls or by bulk driving as in air-fluidized beds. However, given that this way of supplying energy develops in most cases strong spatial gradients, the theoretical description of the above situations is quite complex. Thus, to avoid this problem, it is common in theoretical and computational works to inject energy into the system by the action of external driving forces or *thermostats*. A remarkable observation is that the transport properties of granular systems depend not only on the mechanical properties of the grains but also on the thermostating method. An alternative to the use of thermostats has been proposed in the last few years: the so-called Δ-model [1] where the thermostat is a collisional one since energy is injected in every collision. To be more precise, in a binary collision between particles of species i and j, apart from the usual terms appearing in the collision rules, an extra *constant* velocity Δ_{ij} term is added to the normal component of the relative velocity of the two colliding spheres. Thus, in a binary collision, the change in kinetic energy is constituted by two terms: (i) a dissipation energy term proportional to $1-\alpha_{ij}^2$ and (ii) two energy injection terms with intensity depending on Δ_{ij}. The Δ-model has been mainly proposed to study dynamic properties of granular systems confined in quasi-two-dimensional geometries.

At a kinetic level, the relevant information on the state of the system is provided by the knowledge of the one-particle velocity distribution functions $f_i(\mathbf{r},\mathbf{v};t)$. For moderate densities and in the absence of external forces, the distributions f_i of the Δ-model verify the set of coupled Enskog kinetic equations

$$\frac{\partial}{\partial t}f_i+\mathbf{v}\cdot\nabla f_i=\sum_{j=1}^{s}J_{ij}[f_i,f_j], \qquad (1)$$

where J_{ij} is the Enskog collision operator of the Δ-model [2]. Our main objective here is to solve Eq. (1) by means of the Chapman–Enskog (CE) method for states with small spatial gradients. This allow us to determine the Navier–Stokes transport coefficients of the confined quasi-two-dimensional granular mixture. Before doing it, as a first step we analyze the homogeneous state state (HSS). The study of this state is crucial since its *local* version is the reference state in the Chapman–Enskog solution. As expected, our solution shows that the partial temperatures T_i of each species (measuring its mean kinetic energy) in the HSS are different and hence, energy equipartition is broken down.

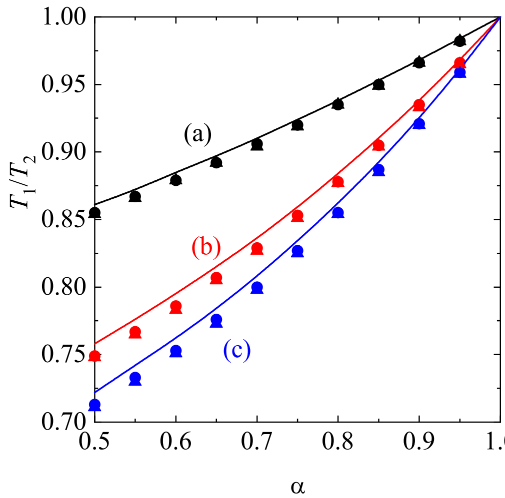

Fig. 1. Plot of T_1/T_2 versus α for $m_1/m_2=\sigma_1/\sigma_2=1$, $\Delta_{22}=\lambda\Delta_{11}$, and $\Delta_{12}=(\Delta_{11}+\Delta_{22})/2$. Here, $\lambda=2$ (a), $\lambda=5$ (b), and $\lambda=10$ (c). Circles are DSMC results while triangles refer to MD simulations for a volume fraction $\phi=0.01$.

As an illustration, Fig. 1 shows T_1/T_2 as a function of the (common) coefficient of restitution $\alpha_{ij}\equiv\alpha$ for a binary mixture $(s=2)$. It is quite apparent that the temperature ratio is clearly different from 1, showing the lack of energy equipartition. We also observe a good agreement between the (approximate) theoretical results (based on the use of Maxwellian distribution to estimate the partial cooling rates) and computer simulations.

Once the HSS is characterized, the next step is to solve Eq. (1) for states near to the HSS. As said before, this solution can be obtained by the application of the CE method. Explicit forms for the diffusion transport coefficients, the shear viscosity coefficient, and the coefficients associated with the heat flux are explicitly obtained in terms of the parameter space of the system by assuming steady state conditions and by considering the leading terms in a Sonine polynomial expansion [3]. As an application, the violation of the Onsager reciprocical relations is quantified.

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