

# Lagrangian betweenness and bottlenecks in ocean flow networks

Enrico Ser-Giacomi<sup>1</sup>, Alberto Baudena<sup>2</sup>, Vincent Rossi<sup>3</sup>, Mick Follows<sup>1</sup>,  
Sophie Clayton<sup>4</sup>, Ruggero Vasile<sup>5,6</sup>, Cristóbal López<sup>7</sup> and Emilio Hernández-García<sup>7</sup>

<sup>1</sup>Department of Earth, Atmospheric and Planetary Sciences, MIT, Cambridge, USA.

<sup>2</sup>Sorbonne Université, Institut de la Mer de Villefranche-sur-Mer, France.

<sup>3</sup>Mediterranean Institute of Oceanography (UM110, UMR 7294), Marseille, France. <sup>4</sup>Old Dominion University, Norfolk, VA, USA.

<sup>5</sup>UP Transfer GmbH, Potsdam, Germany. <sup>6</sup>GFZ German Research Centre for Geosciences, Potsdam, Germany.

<sup>7</sup>IFISC (CSIC-UIB), Instituto de Física Interdisciplinar y Sistemas Complejos, Palma de Mallorca, Spain.

The study of connectivity in networks has brought insights across many fields ranging from neurosciences to epidemic spreading or climate. One of the classical network measures, betweenness centrality, has demonstrated to be very effective in identifying nodes that act as focus of congestion, or bottlenecks. Outside the network framework, however, there is no obvious way to define betweenness. Nevertheless, the concept of bottleneck is equally present in dynamical systems and in fluid flows as in networks (see Fig. 2).

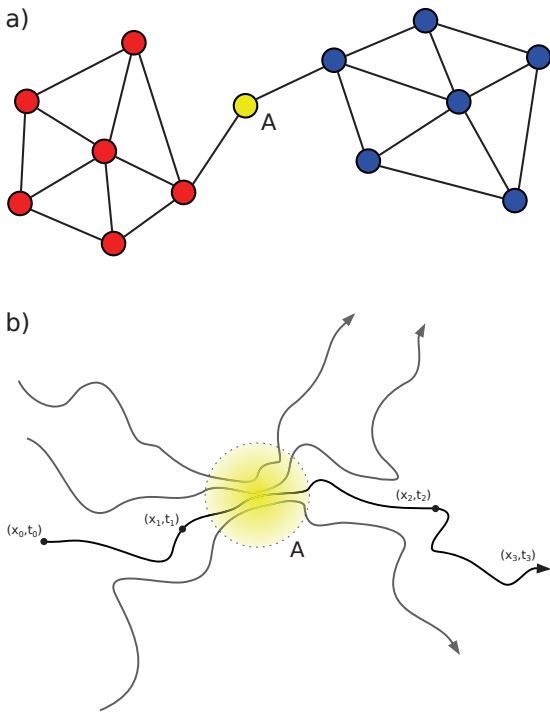


Fig. 1. Regions marked as ‘A’ have large betweenness in the network of panel a), and also in the dynamical system or fluid flow in b), representing in both cases a *bottleneck*.

By using ideas that relate dynamical systems and network theory [1], we have provided a trajectory-based formulation of betweenness, called Lagrangian betweenness [2], which is computed in terms of Lyapunov exponents. This extends the concept of betweenness beyond network theory and relates hyperbolic points and heteroclinic connections in dynamical system to the structural bottlenecks of the flow network associated with it.

We illustrate the use and meaning of the Lagrangian betweenness by identifying bottlenecks in ocean surface flows

in the Adriatic sea or the Kerguelen region in the Southern Ocean. Also, by analyzing plankton abundance data from the Kuroshio region in the Pacific Ocean, we find significant spatial correlations between measures of biological diversity and betweenness, suggesting that ocean bottlenecks act as ecological hot spots.

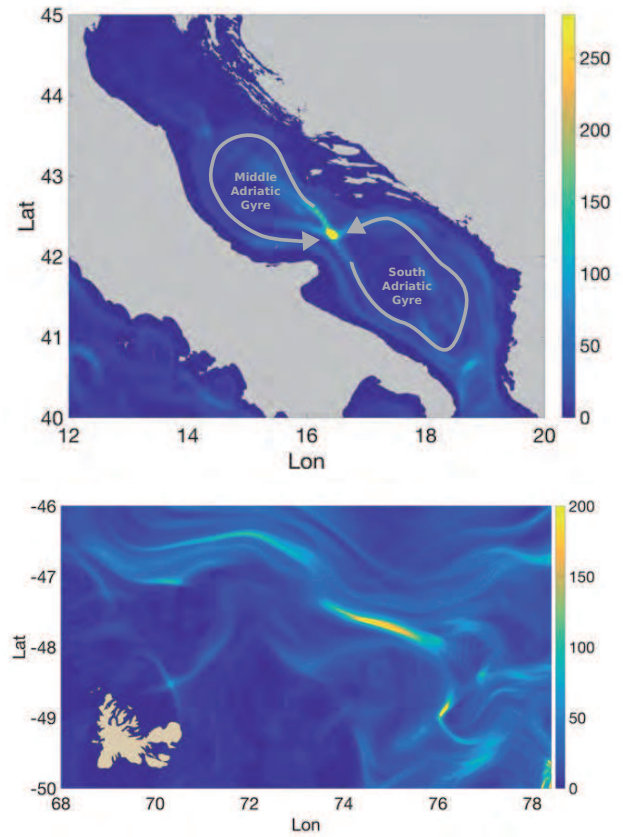


Fig. 2. Two examples of betweenness field in the Adriatic sea (upper panel) and in the Kerguelen region (lower panel). Large values (yellow color) identify flow bottlenecks.

[1] E. Ser-Giacomi, V. Rossi, C. López, and E. Hernández-García, *Flow networks: A characterization of geophysical fluid transport*, *Chaos* **25**, 036404 (2015).

[2] E. Ser-Giacomi, A. Baudena, V. Rossi, M. Follows, S. Clayton, R. Vasile, C. López and E. Hernández-García, *Lagrangian betweenness as a measure of bottlenecks in dynamical systems with oceanographic examples*, *Nature Communications* **12**, 4935 (2021).