The Biased voter model: How persuasive a small group can be?

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In this work, we study a variation of the classical voter model, where voters have a biased constant confidence. To have confidence, means that with a constant probability p they keep their opinion upon an interaction with their neighbor, instead of copying. We assume that the default confidence is $p = 1/2$. However for a fraction γ of these voters we assume that are biased towards a fixed opinion, in the sense that their confidence when changing from a fixed state, say $s_i = -1$ to $s_i = +1$ is given by $p = (1+v)/2$ with v being the bias parameter, while the reverse switch $s_i = +1$ to $s_i = -1$ occurs with a confidence equal to $p = (1-v)/2$. This setup resembles that of [1], where however all voters were biased and only interactions on a complete graph (CG) were studied. We considered two distinct scenarios in our studies. First, we assumed that there was no dependence of the topology of the network on which the dynamics took place and the type of voters, in which case we studied the model on the CG as well as on an Erdős-Rényi (ER) network. Then we assumed that the topology of interactions of the two distinct type of voters was indeed dependent on their type and we examined strategies that the biased voters could follow to convince faster the rest of the voters to adopt their opinion.

For the biased-independent topology of interactions, we initially focused the study on the thermodynamic limit of the system for both the complete graph as well as the Erdős-Rényi (ER) network. In both cases, by considering the relevant Fokker-Planck equations, we showed that in general, at the long time limit, contrary to the standard voter model, the magnetization is not constant, but rather tends to $m = 1$, with an active link density $\rho = 0$. This happens at a time of the order of $\frac{1}{\gamma v}$.

Moving beyond this, we considered the finite size effects. To solve the dynamics in the case of the CG we resorted to a mean field approach, where for the ER case we made use of the pair approximation. In addition, for the ER case, we made an adiabatic approximation, i.e. we assumed that the magnetization's dynamics were slowly changing in comparison to that of the active link density and hence the latter followed the dynamics of the former. This allowed us to obtain the dependence of the active link density on the magnetization. We focused mainly on two observables, the fixation probability $P_1(\sigma)$, or probability to reach the preferred state at a finite number of steps, and the consensus time $\tau(\sigma)$ for which we derive analytical expressions. In particular for the ER case, we obtain specifically, for the fixation probability $P_1 = \frac{1}{1+e^{-\tilde{\beta}/2}}$, where $\tilde{\beta} = 2\gamma v N$ for the CG and $\tilde{\beta} = 2\gamma v \frac{\mu}{1+\mu} N$ for the ER network, with N being the number of voters. Furthermore, we find that for the special case of balanced initial condition $\sigma = 1/2$ and sufficiently large N the time to reach consensus, τ scales as $\tilde{B}\log(N)/(\gamma v)$ for both the CG and the ER network, where $\tilde{B} = 1$ for the former and $\tilde{B} = \frac{\mu-1-\gamma v}{\mu-2}$ for the latter. This is to be contrasted to the case of the classical voter model where for both the CG and the ER network scales linearly with N [2].

Finally, we also study the case where the voters lie on biased-dependent heterogeneous Erdős-Rényi networks. In this case, we defined as the parameter that quantifies our deviation from the homogeneous random network, the ratio $\delta = \frac{\mu_{BB}}{\mu_{UU}}$, where μ_{XY} represents the average degree of connections between voters of type X to voters of type Y. With this in mind we identified two candidate total average degree μ preserving strategies with which one can affect the time to consensus by varying δ. In strategy I we considered the case of varying μ_{BU} at the expense of μ_{BB} and μ_{UU}. We found that the consensus time is indifferent to this variation. On the contrary in strategy II we considered the scenario of varying μ_{BB} (and accordingly μ_{UU}) while keeping μ_{BU} constant. We found that increasing μ_{BB} resulted in a significant reduction of the consensus time τ, as well as to a significant increment to the probability of reaching consensus to the preferred state. The main conclusion of our studies is then that the more endogamous a community of biased voters is, where we define an endogamous community as one where its members have a higher average degree compared to that of the rest of the members of the ER network, the faster it can lead the rest of the society to agreement to their preferred state.

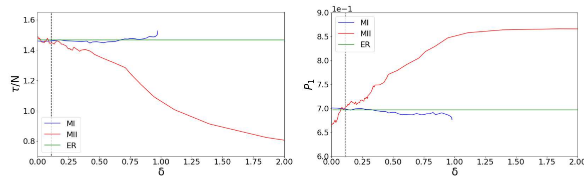

Fig. 1. In the left (right) figure, the green line represents the consensus time (fixation probability) obtained analytically for a homogeneous ER network of a given average degree μ. In both figures, the blue (red) line represents for the same μ, the scenario of the dynamics on a biased-dependent heterogeneous ER where δ varies and provokes topology changes according to model I (II). The dashed line is the value δ_{random} for the homogeneous topology scenario. The main result, is that for model II a significant decrease (increase) in consensus time (fixation probability) is observed. This is the strategy, in terms of the topology of interactions, that the small group has to follow to convince the rest of the society faster. Here $N = 1000$, $\gamma = 0.1$, $v = 0.01$ and $\sigma_0 = 0.5$

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[2] F. Vazquez and V. M. Eguluz, *Analytical solution of the voter model on uncorrelated networks* (New Journal of Physics, 10(6):63011, 2008).