Mapping of First-Passage Percolation into bond percolation under extreme disorder

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Geometry on random manifolds presents both applied and fundamental interest, with applications ranging from the physics of polymers and membranes to quantum gravity. It was recently shown that, in the case of random surfaces which are flat in average and with short-range correlations in the curvature, geodesics present fractal structure, governed by exponents corresponding to the Kardar-Parisi-Zhang universality class (KPZ). When the manifold is discretized the problem is called First-Passage Percolation (FPP).

The FPP model consists of an undirected lattice where a link-time t is assigned randomly to each edge between neighboring nodes. Link-times are independent and identically distributed positive random numbers with common cumulative distribution function $F(t)$. The principal object of study in FPP are geodesics, i.e. the minimal-time paths joining pairs of nodes, and balls $B(T)$ given by the set of nodes which can be reached from the origin in a time less than the passage time T. When the lattice structure is smooth, the ball $B(T)$ grows linearly with T and has a non-random asymptotic shape. However when the probability function is properly tuned to introduce a high level of noise in the link-time distribution, the shape of the ball becomes completely irregular (Fig.1).

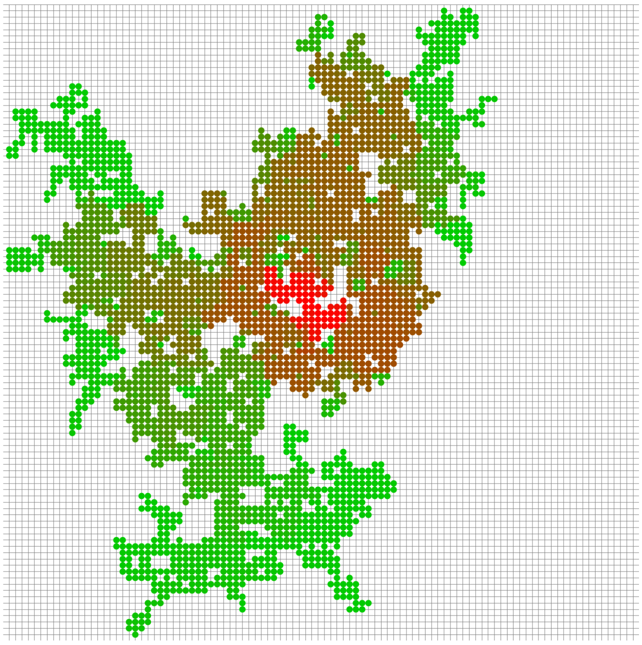

Fig. 1. Geodesic ball from First-Passage Percolation under strong disorder. Different colors represent different values of the passage time T; they correspond to different steps of the ball's growth.

In a previous work [1] we analyzed the statistical properties of arrival times to FPP in weak-disorder regime and we showed a crossover between Gaussian and KPZ universality, with the crossover length decreasing as the disorder grows. In this work [2] we have gone one step further by considering the strong-disorder regime, where a new characteristic length appears below which the model displays bond-percolation universality class. In our work we provide a thorough characterization of the bond-percolation phase reproducing its critical exponents through a scaling analysis of the balls, for three different distributions: Weibull, Pareto and Log-Normal. The key to do this is the continuous mapping of the FPP passage time T into the occupation probability p of the bond-percolation problem:

$$p = F(T). \tag{1}$$

The behavior of the new characteristic length can be explained from the properties of the link-time distribution. Moreover, the interplay between the correlation length intrinsic to percolation and this new characteristic length determines the crossover between initial percolation-like growth and asymptotic KPZ scaling. We also provide a first study of the behavior of this interplay.

As long as the new characteristic length stays above the correlation length intrinsic to percolation, we were allowed to observe percolation-like growth in FPP models. But when the correlation length of percolation overtakes the FPP characteristic length, the model starts to develop KPZ scaling.

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[1] P. Córdoba-Torres, S. N. Santalla, R. Cuerno and J. Rodríguez-Laguna, *Kardar-Parisi-Zhang universality in first passage percolation: the role of geodesic degenerancy*, J. Stat. Mech 063212 (2018).

[2] D. Villarrubia, I. A. Domenech, S. N. Santalla, J. Rodríguez-Laguna and P. Córdoba-Torres, *First-Passage Percolation under extreme disorder: From bond percolation to Kardar-Parisi-Zhang universality*, Phys. Rev. E **101**, 062124 (2020).